THE GENERAL SINE FUNCTION

The **general sine function** is

$$y = a\sin(b(x - c)) + d \quad \text{where} \ \ b > 0.$$

affects   affects   affects   affects
amplitude   period   horizontal translation   vertical translation

The **principal axis** of the general sine function is $y = d$.

The **period** of the general sine function is $\dfrac{2\pi}{b}$.

The **amplitude** of the general sine function is $|a|$.

Consider $y = 2\sin(3(x - \frac{\pi}{4})) + 1$. It is a translation of $y = 2\sin 3x$ with translation vector $\begin{pmatrix} \frac{\pi}{4} \\ 1 \end{pmatrix}$.

So, starting with $y = \sin x$ we would:

- double the amplitude to produce $y = 2\sin x$,   then
- divide the period by 3 to produce $y = 2\sin 3x$,   then
- translate by $\begin{pmatrix} \frac{\pi}{4} \\ 1 \end{pmatrix}$ to produce $y = 2\sin(3(x - \frac{\pi}{4})) + 1$.

EXERCISE 10B.2

1 Sketch the graphs of the following for $0 \leqslant x \leqslant 4\pi$:

   a $y = \sin x - 2$ **b** $y = \sin(x - 2)$

   c $y = \sin(x + 2)$ **d** $y = \sin x + 2$

   e $y = \sin(x + \frac{\pi}{4})$ **f** $y = \sin(x - \frac{\pi}{6}) + 1$

GRAPHING PACKAGE

Check your answers using technology.

2 State the period of:

   a $y = \sin 5t$ **b** $y = \sin\left(\frac{t}{4}\right)$ **c** $y = \sin(-2t)$

3 Find b in $y = \sin bx$ if $b > 0$ and the period is:

   a 3π **b** $\frac{\pi}{10}$ **c** 100π **d** 50

4 State the transformation(s) which map:

   a $y = \sin x$ onto $y = \sin x - 1$ **b** $y = \sin x$ onto $y = \sin(x - \frac{\pi}{4})$

   c $y = \sin x$ onto $y = 2\sin x$ **d** $y = \sin x$ onto $y = \sin 4x$

   e $y = \sin x$ onto $y = \frac{1}{2}\sin x$ **f** $y = \sin x$ onto $y = \sin\left(\frac{x}{4}\right)$

   g $y = \sin x$ onto $y = -\sin x$ **h** $y = \sin x$ onto $y = -3 + \sin(x + 2)$

   i $y = \sin x$ onto $y = 2\sin 3x$ **j** $y = \sin x$ onto $y = \sin(x - \frac{\pi}{3}) + 2$