5 **a** Length of arc AB $= \theta \times$ radius

$\therefore\ \pi = \theta \times 3$

$\therefore\ \theta = \frac{\pi}{3}$

b Area of circle $= \pi r^2$

$= \pi \times 3^2$

$= 9\pi \text{ cm}^2$

Area of sector $= \frac{1}{2}\theta r^2$

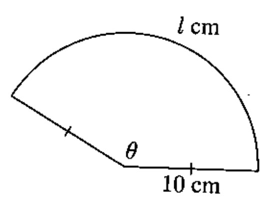
$= \frac{1}{2} \times \frac{\pi}{3} \times 3^2$

$= \frac{3\pi}{2} \text{ cm}^2$

$\therefore$ shaded area $= 9\pi - \frac{3\pi}{2}$

$= \frac{15\pi}{2} \text{ cm}^2$

6



a perimeter $= 40$ cm

$\therefore\ 10 + 10 + l = 40$

$\therefore\ l = 20$

So, the arc length is 20 cm.

b area $= \frac{1}{2}\theta r^2$

$= \frac{1}{2}lr \quad \{l = \theta r\}$

$= \frac{1}{2} \times 20 \times 10$

$= 100 \text{ cm}^2$

7 **a** area $= 20 \text{ cm}^2$

$\therefore\ \frac{1}{2}\theta r^2 = 20$

$\therefore\ \frac{1}{2}lr = 20 \quad \{l = \theta r\}$

$\therefore\ \frac{1}{2}(6)r = 20$

$\therefore\ r = \frac{20}{3}$ cm

b $\theta = \frac{l}{r}$

$= \frac{6}{\frac{20}{3}}$

$= 0.9$

8 **a** Length of arc PQ $= \frac{\theta}{360} \times 2\pi r$

$= \frac{135}{360} \times 2\pi \times 4$

$= 3\pi$ m

b Area of whole sector POQ $= \frac{\theta}{360} \times \pi r^2$

$= \frac{135}{360} \times \pi \times 4^2$

$= 6\pi \text{ m}^2$

Area of unshaded sector $= \frac{135}{360} \times \pi \times 2^2$

$= \frac{3\pi}{2} \text{ m}^2$

So, shaded area $= 6\pi - \frac{3\pi}{2}$

$= \frac{9\pi}{2} \text{ m}^2$

9 **a** $\cos^2(\frac{\pi}{4}) - \sin^2(\frac{5\pi}{6}) = (\frac{1}{\sqrt{2}})^2 - (\frac{1}{2})^2$

$= \frac{1}{2} - \frac{1}{4}$

$= \frac{1}{4}$

b $\tan 60° = \sqrt{3}$

$\therefore$ the equation is $y = \sqrt{3}x$

10 **a** $\tan(-\frac{\pi}{6}) - \cos(\frac{4\pi}{3}) = -\frac{1}{\sqrt{3}} - (-\frac{1}{2})$

$= \frac{1}{2} - \frac{\sqrt{3}}{3}$

b $\tan(\frac{2\pi}{3}) = -\sqrt{3}$

$\therefore$ the equation is

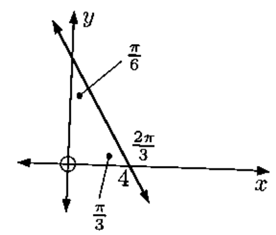
$y = -\sqrt{3}x + c$

But when $x = 4$, $y = 0$

$\therefore\ 0 = -4\sqrt{3} + c$

$\therefore\ c = 4\sqrt{3}$

So, $y = -\sqrt{3}x + 4\sqrt{3}$

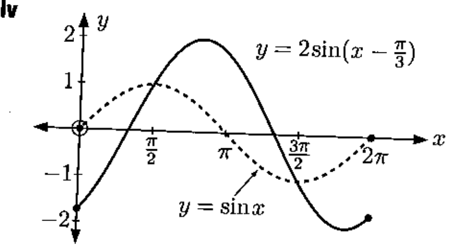


11 **a** **i** amplitude $= 2$

ii principal axis is $y = 0$

iii period $= \frac{2\pi}{1} = 2\pi$

iv

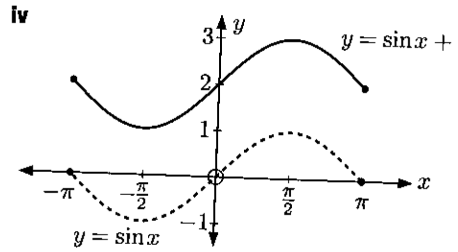


b **i** amplitude $= 1$

ii principal axis is $y = 2$

iii period $= \frac{2\pi}{1} = 2\pi$

iv

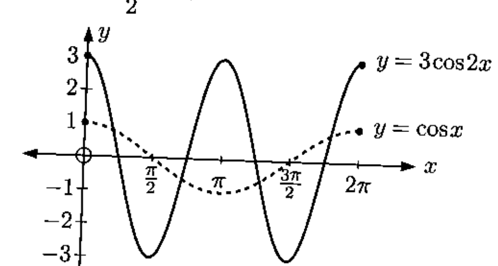


c **i** amplitude $= 3$

ii principal axis is $y = 0$

iii period $= \frac{2\pi}{2} = \pi$

iv

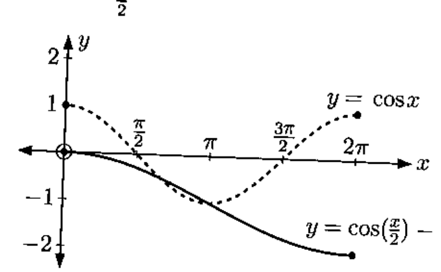


d **i** amplitude $= 1$

ii principal axis is $y = -1$

iii period $= \frac{2\pi}{\frac{1}{2}} = 4\pi$

iv

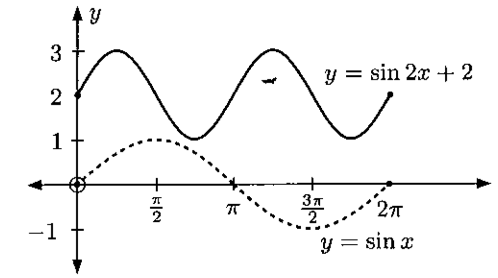


e **i** amplitude $= 1$

ii principal axis is $y = 2$

iii period $= \frac{2\pi}{2} = \pi$

iv

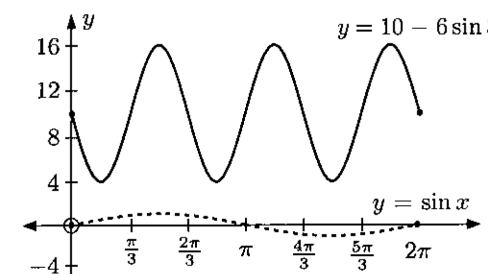


f **i** amplitude $= |-6| = 6$

ii principal axis is $y = 10$

iii period $= \frac{2\pi}{3}$

iv

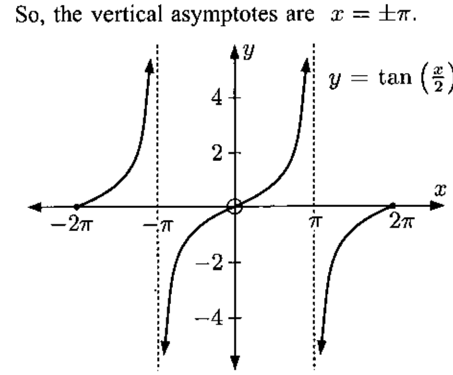


12 **a** **i** period $= \frac{\pi}{\frac{1}{2}} = 2\pi$

ii $y = \tan(\frac{x}{2})$ is a horizontal stretch of $y = \tan x$ with scale factor 2.

So, the vertical asymptotes are $x = \pm\pi$.

iii



b **i** period $= \frac{\pi}{3}$

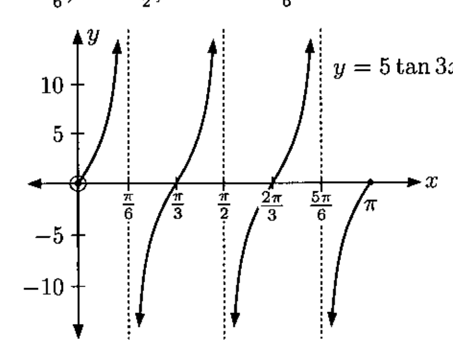
ii $y = 5\tan 3x$ is undefined when $\cos 3x = 0$

$\therefore\ 3x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$

$\therefore\ x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \ldots$

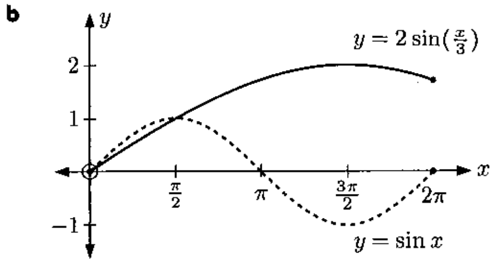
So, for the given domain, the vertical asymptotes are $x = \frac{\pi}{6}$, $x = \frac{\pi}{2}$, and $x = \frac{5\pi}{6}$.

iii



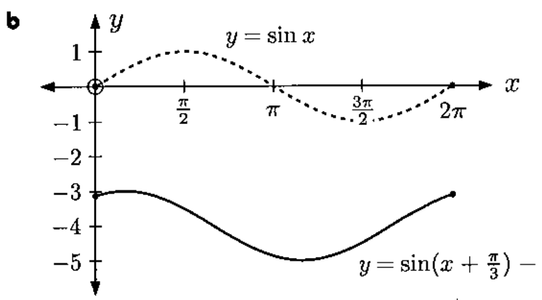
13 **a** A vertical stretch with scale factor 2, and a horizontal stretch with scale factor 3.

b



14 **a** A translation of $\frac{\pi}{3}$ units to the left, and a translation of 4 units downwards.

b



15 **a** $y(0) = 4$ so $a + b\sin 0 = 4$

$\therefore\ a = 4$

b $y(\frac{\pi}{2}) = 1$

$\therefore\ 4 + b\sin(\frac{\pi}{2}) = 1$

$\therefore\ b(1) = -3$

$\therefore\ b = -3$

Check: $y = 4 - 3\sin x$

$y(\pi) = 4 - 3\sin\pi = 4 - 0 = 4$ ✓

16 Period $= \frac{2\pi}{b} = \pi$, so $b = 2$

Amplitude is 10, so $a = 10$.

Principal axis is $y = 15$, so $c = 15$.

$\therefore\ y = 10\sin 2x + 15$

Check: $y(\frac{\pi}{4}) = 10\sin(\frac{\pi}{2}) + 15$

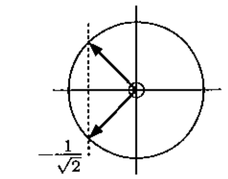
$= 10(1) + 15$

$= 25$ ✓

17 **a** $\sqrt{2}\cos x + 1 = 0$

$\therefore\ \cos x = -\frac{1}{\sqrt{2}}$

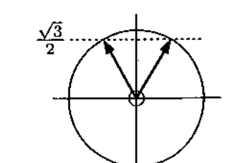
$\therefore\ x = \frac{3\pi}{4}$ or $\frac{5\pi}{4}$



b $2\sin x = \sqrt{3}$

$\therefore\ \sin x = \frac{\sqrt{3}}{2}$

$\therefore\ x = \frac{\pi}{3}$ or $\frac{2\pi}{3}$



c $2\sin^2 x + 3\cos x = 3$

$2(1 - \cos^2 x) + 3\cos x - 3 = 0$

$\therefore\ 2 - 2\cos^2 x + 3\cos x - 3 = 0$

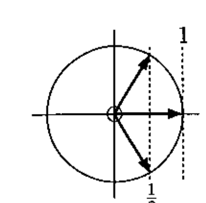
$\therefore\ -2\cos^2 x + 3\cos x - 1 = 0$

$\therefore\ 2\cos^2 x - 3\cos x + 1 = 0$

$\therefore\ (2\cos x - 1)(\cos x - 1) = 0$

$\therefore\ \cos x = \frac{1}{2}$ or 1

$\therefore\ x = 0, \frac{\pi}{3}, \frac{5\pi}{3}, 2\pi$



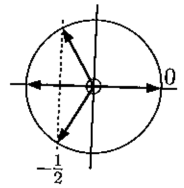
d $\sin 2x + \sin x = 0$

∴ $2\sin x\cos x + \sin x = 0$

∴ $\sin x(2\cos x + 1) = 0$

∴ $\sin x = 0$ or $\cos x = -\frac{1}{2}$

∴ $x = 0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, 2\pi$

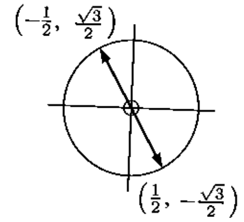


e $\sin x = -\sqrt{3}\cos x$

∴ $\frac{\sin x}{\cos x} = -\sqrt{3}$

∴ $\tan x = -\sqrt{3}$

∴ $x = \frac{2\pi}{3}, \frac{5\pi}{3}$



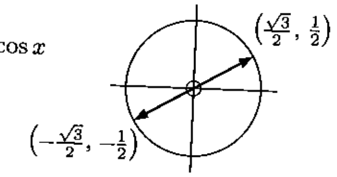
f $\frac{1}{\sqrt{3}}\cos x - \sin x = 0$

∴ $\sin x = \frac{1}{\sqrt{3}}\cos x$

∴ $\frac{\sin x}{\cos x} = \frac{1}{\sqrt{3}}$

∴ $\tan x = \frac{1}{\sqrt{3}}$

∴ $x = \frac{\pi}{6}, \frac{7\pi}{6}$



18 **a** $y = a\cos(b(x-c)) + d$

i The amplitude is 5, so $a = 5$.

ii The period is 4, so $\frac{2\pi}{b} = 4$

∴ $b = \frac{\pi}{2}$

iii The principal axis is $y = 2$, so $d = 2$.

iv So far, the curve is $y = 5\cos(\frac{\pi}{2}(x-c)) + 2$

A maximum occurs when $x = 4$.

∴ $\frac{\pi}{2}(4-c)$ is a multiple of 2π

∴ we can choose $c = 0$.

b From **a**, $y = 5\cos(\frac{\pi}{2}x) + 2$

Since $\cos\theta = \sin(\theta + \frac{\pi}{2})$, the sine function of this curve is $y = 5\sin(\frac{\pi}{2}x + \frac{\pi}{2}) + 2$.

19 **a** $\sin^2\theta + \cos^2\theta = 1$

∴ $\sin^2\theta + \frac{9}{64} = 1$

∴ $\sin^2\theta = \frac{55}{64}$

∴ $\sin\theta = \pm\frac{\sqrt{55}}{8}$

∴ $\sin\theta = \frac{\sqrt{55}}{8}$ {θ is acute}

b $\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{\sqrt{55}}{8}\right)\left(\frac{3}{8}\right)$

$= \frac{3}{32}\sqrt{55}$

20 **a** $\cos^2\alpha + \sin^2\alpha = 1$

∴ $\cos^2\alpha + \frac{4}{9} = 1$

∴ $\cos^2\alpha = \frac{5}{9}$

∴ $\cos\alpha = \pm\frac{\sqrt{5}}{3}$

∴ $\cos\alpha = -\frac{\sqrt{5}}{3}$ {α is obtuse}

b $\cos 2\alpha = 1 - 2\sin^2\alpha$

$= 1 - 2\left(\frac{2}{3}\right)^2$

$= 1 - \frac{8}{9}$

$= \frac{1}{9}$

21 **a** $\cos 2\alpha = \frac{5}{13}$

∴ $1 - 2\sin^2\alpha = \frac{5}{13}$

∴ $2\sin^2\alpha = \frac{8}{13}$

∴ $\sin^2\alpha = \frac{4}{13}$

∴ $\sin\alpha = \pm\frac{2}{\sqrt{13}}$

But α is acute and so $\sin\alpha > 0$

∴ $\sin\alpha = \frac{2}{\sqrt{13}}$

b $\cos^2\alpha + \sin^2\alpha = 1$

∴ $\cos^2\alpha + \frac{4}{13} = 1$ {from **a**}

∴ $\cos^2\alpha = \frac{9}{13}$

∴ $\cos\alpha = \pm\frac{3}{\sqrt{13}}$

∴ $\cos\alpha = \frac{3}{\sqrt{13}}$ {α is acute}

c $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$

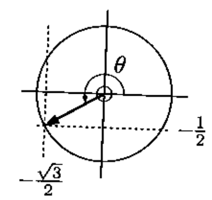
$= \frac{\frac{2}{\sqrt{13}}}{\frac{3}{\sqrt{13}}}$

$= \frac{2}{3}$

22 **a** $\theta = \frac{7\pi}{6}$

b $\tan\theta = \frac{\sin\theta}{\cos\theta}$

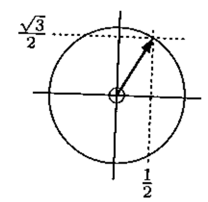
$= \frac{1}{\sqrt{3}}$



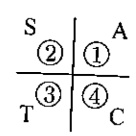
c Since $\theta = \frac{7\pi}{6}$,

$2\theta = \frac{7\pi}{3}$

∴ $\tan 2\theta = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$



23 **a**



$\tan\theta > 0$ and $\cos\theta < 0$ in the third quadrant.

b $\tan\theta = \frac{\sin\theta}{\cos\theta} = 2$

∴ $\sin\theta = 2\cos\theta$

Now $\cos^2\theta + \sin^2\theta = 1$

∴ $\cos^2\theta + (2\cos\theta)^2 = 1$

∴ $5\cos^2\theta = 1$

∴ $\cos\theta = \pm\frac{1}{\sqrt{5}}$

But $\cos\theta < 0$ in Q3

∴ $\cos\theta = -\frac{1}{\sqrt{5}}$

c $y = \tan(x + \frac{\pi}{6}) + 2$ is a translation of $y = \tan x$ through $\begin{pmatrix} -\frac{\pi}{6} \\ 2 \end{pmatrix}$.

24 **a** For the sine function $y = a\sin b(x-c) + d$:

- the amplitude $= 2$, so $a = 2$
- the period $= \pi$, so $\frac{2\pi}{b} = \pi$ ∴ $b = 2$
- the principal axis is $y = 1$, so $d = 1$
- there is no horizontal translation, so $c = 0$.

b The function is $y = 2\sin 2x + 1$.

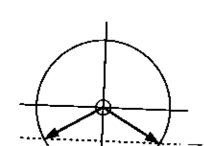
∴ we need to solve $2\sin 2x + 1 = 0$, $0 \leqslant x \leqslant \pi$

∴ $\sin 2x = -\frac{1}{2}$

∴ $2x = \frac{7\pi}{6}$ or $\frac{11\pi}{6}$

∴ $x = \frac{7\pi}{12}$ or $\frac{11\pi}{12}$

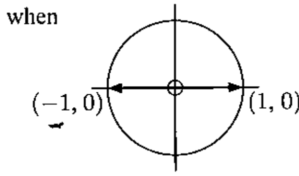
So, P is $(\frac{7\pi}{12}, 0)$ and Q is $(\frac{11\pi}{12}, 0)$.



25 **a** $f(x) = \frac{1}{\sin x}$ is defined when

$\sin x \neq 0$

∴ $x \neq 0 + k\pi$



So, the domain is $\{x \mid x \neq k\pi,\ k \in \mathbb{Z}\}$

Since the range of $\sin x$ is $\{y \mid -1 \leqslant y \leqslant 1\}$, the values of $\frac{1}{\sin x}$ are $\leqslant -1$ or $\geqslant 1$.

So, the range is $\{y \mid y \leqslant -1$ or $y \geqslant 1\}$.

b $f(x)$ is undefined when $x = k\pi$, $k \in \mathbb{Z}$

So, $x = 0$, $x = \pm\pi$, $x = \pm 2\pi$, ...

are all vertical asymptotes.

26 **a** $\tan\theta = \frac{\sin\theta}{\cos\theta}$ ∴ $\cos\theta = \frac{\sin\theta}{\tan\theta} = \frac{-\frac{4}{5}}{-\frac{4}{3}} = \frac{3}{5}$

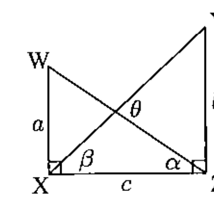
b $\cos 2\theta = \cos^2\theta - \sin^2\theta = (\frac{3}{5})^2 - (-\frac{4}{5})^2$

$= -\frac{7}{25}$

27 We label points W, X, Y and Z as shown, and let

$\mathrm{Y\widehat{X}Z} = \beta$ and

$\mathrm{W\widehat{Z}X} = \alpha$.



Now $\tan\beta = \frac{b}{c}$ {from $\triangle$XYZ}

∴ $\beta = \tan^{-1}\left(\frac{b}{c}\right)$

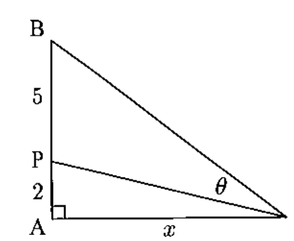
Likewise, $\tan\alpha = \frac{a}{c}$ {from $\triangle$XWZ}

∴ $\alpha = \tan^{-1}\left(\frac{a}{c}\right)$

Now $\theta = \alpha + \beta$ {external angle of a triangle}

∴ $\theta = \tan^{-1}\left(\frac{a}{c}\right) + \tan^{-1}\left(\frac{b}{c}\right)$ as required.

28



$\mathrm{B\widehat{C}A} = \tan^{-1}\left(\frac{7}{x}\right)$ and $\mathrm{P\widehat{C}A} = \tan^{-1}\left(\frac{2}{x}\right)$

Now $\theta = \mathrm{B\widehat{C}A} - \mathrm{P\widehat{C}A}$

∴ $\theta = \tan^{-1}\left(\frac{7}{x}\right) - \tan^{-1}\left(\frac{2}{x}\right)$

29 **a** $\cos 2x = \frac{5}{8}$

∴ $1 - 2\sin^2 x = \frac{5}{8}$ {double angle formula}

∴ $2\sin^2 x = \frac{3}{8}$

∴ $\sin x = \pm\frac{\sqrt{3}}{4}$

∴ $\sin x = \frac{\sqrt{3}}{4}$ {x is acute}

b $\sin^2 x + \cos^2 x = 1$

∴ $(\frac{\sqrt{3}}{4})^2 + \cos^2 x = 1$

∴ $\frac{3}{16} + \cos^2 x = 1$

∴ $\cos^2 x = \frac{13}{16}$

∴ $\cos x = \pm\frac{\sqrt{13}}{4}$

∴ $\cos x = \frac{\sqrt{13}}{4}$ {x is acute}

Now, $\sin 2x = 2\sin x\cos x$

$= 2(\frac{\sqrt{3}}{4})(\frac{\sqrt{13}}{4})$

$= \frac{\sqrt{39}}{8}$

∴ $\tan 2x = \frac{\sin 2x}{\cos 2x}$

$= \frac{\frac{\sqrt{39}}{8}}{\frac{5}{8}}$

$= \frac{\sqrt{39}}{5}$

30 **a** $\sin^2\theta + \cos^2\theta = 1$

∴ $\left(\frac{2}{3}\right)^2 + \cos^2\theta = 1$

∴ $\cos^2\theta = \frac{5}{9}$

∴ $\cos\theta = -\frac{\sqrt{5}}{3}$ {θ is obtuse}

$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2 \times \frac{2}{3} \times -\frac{\sqrt{5}}{3}$

$= -\frac{4\sqrt{5}}{9}$

b $\cos 2\theta = 1 - 2\sin^2\theta$

$= 1 - 2(\frac{2}{3})^2$

$= 1 - 2(\frac{4}{9})$

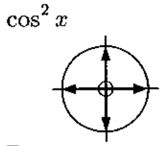
$= \frac{1}{9}$

31 $(\sin x + \cos x)^2 = \sin^2 x + \cos^2 x$

∴ $\sin^2 x + 2\sin x\cos x + \cos^2 x = \sin^2 x + \cos^2 x$

∴ $2\sin x\cos x = 0$

∴ $\sin x = 0$ or $\cos x = 0$



Since $0 \leqslant x \leqslant \pi$, $x = 0, \frac{\pi}{2}$, or π.

32 $\sin^2 t + \cos^2 t = 1$,

∴ $\sin^2 t = 1 - \cos^2 t$

For $0 < t < \pi$, $\sin t > 0$

∴ $\sin t = \sqrt{1 - \cos^2 t}$

∴ $\tan t = \frac{\sin t}{\cos t} = \frac{\sqrt{1-\cos^2 t}}{\cos t}$ for $0 < t < \pi$

33 **a** $\sin^{-1}(3x^2 - 2x) = \frac{5\pi}{6}$

∴ $3x^2 - 2x = \sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$

∴ $6x^2 - 4x - 1 = 0$

∴ $x = \frac{4 \pm \sqrt{(-4)^2 - 4 \times 6(-1)}}{2 \times 6}$

$= \frac{4 \pm \sqrt{40}}{12} = \frac{2 \pm \sqrt{10}}{6}$

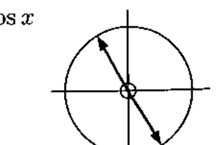
b $\sin x + \sqrt{3}\cos x = 0$

∴ $\sin x = -\sqrt{3}\cos x$

∴ $\frac{\sin x}{\cos x} = -\sqrt{3}$

∴ $\tan x = -\sqrt{3}$

∴ for $0 \leqslant x \leqslant 2\pi$, $x = \frac{2\pi}{3}, \frac{5\pi}{3}$



34 **a** $\frac{1 - \cos 2\theta}{\sin 2\theta} = \frac{1 - (1 - 2\sin^2\theta)}{2\sin\theta\cos\theta}$

$= \frac{2\sin^2\theta}{2\sin\theta\cos\theta}$

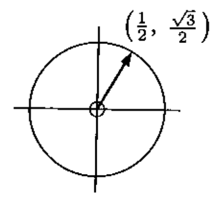
$= \frac{\sin\theta}{\cos\theta}$

$= \tan\theta$

b $\dfrac{1-\cos 2\theta}{\sin 2\theta} = \sqrt{3}$

$\therefore\ \tan\theta = \sqrt{3}$

$\therefore\ \theta = \frac{\pi}{3}$ as $0 < \theta < \frac{\pi}{2}$



35 **a** $\dfrac{\sin^2\theta}{1+\cos\theta} = \dfrac{1-\cos^2\theta}{1+\cos\theta}$

$= \dfrac{(1+\cos\theta)(1-\cos\theta)}{(1+\cos\theta)}$

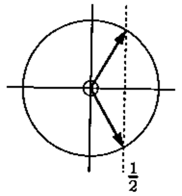
$= (1-\cos\theta)$ provided $\cos\theta \neq -1$

b $\dfrac{\sin^2\theta}{1+\cos\theta} = \frac{1}{2}$

$\therefore\ 1-\cos\theta = \frac{1}{2}$

$\therefore\ \cos\theta = \frac{1}{2}$

$\therefore\ \theta = -\frac{\pi}{3}, \frac{\pi}{3}$



36 **a** $-3\cos 2\theta - 14\sin\theta + 11$

$= -3(1-2\sin^2\theta) - 14\sin\theta + 11$

$= -3 + 6\sin^2\theta - 14\sin\theta + 11$

$= 6\sin^2\theta - 14\sin\theta + 8$

b $-3\cos 2\theta - 14\sin\theta + 11 = 0$

$\therefore\ 6\sin^2\theta - 14\sin\theta + 8 = 0$

$\therefore\ 3\sin^2\theta - 7\sin\theta + 4 = 0$

$\therefore\ (3\sin\theta - 4)(\sin\theta - 1) = 0$

$\therefore\ \sin\theta = 1$ $\{-1 \leqslant \sin\theta \leqslant 1\}$

$\therefore\ \theta = \frac{\pi}{2}$

37 **a** $\cos 2\theta + 2\sqrt{2}\cos\theta - 2$

$= (2\cos^2\theta - 1) + 2\sqrt{2}\cos\theta - 2$

$= 2\cos^2\theta - 1 + 2\sqrt{2}\cos\theta - 2$

$= 2\cos^2\theta + 2\sqrt{2}\cos\theta - 3$ as required

b $\cos 2\theta + 2\sqrt{2}\cos\theta - 2 = 0$

$\therefore\ 2\cos^2\theta + 2\sqrt{2}\cos\theta - 3 = 0$

Using the quadratic formula,

$\cos\theta = \dfrac{-2\sqrt{2} \pm \sqrt{(2\sqrt{2})^2 - 4\times 2\times(-3)}}{2\times 2}$

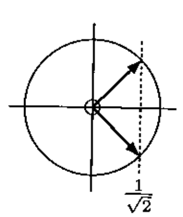
$= \dfrac{-2\sqrt{2} \pm \sqrt{32}}{4}$

$= -\frac{\sqrt{2}}{2} \pm \frac{4\sqrt{2}}{4}$

$= -\frac{1}{\sqrt{2}} \pm \frac{2}{\sqrt{2}}$

$= \frac{1}{\sqrt{2}}$ or $-\frac{3}{\sqrt{2}}$

$\therefore\ \theta = -\frac{\pi}{4}, \frac{\pi}{4}$ $\{-1 \leqslant \cos\theta \leqslant 1\}$



38 **a** $\tan 2A = \sin A$

$\therefore\ \dfrac{\sin 2A}{\cos 2A} = \sin A$

$\therefore\ \dfrac{2\sin A\cos A}{2\cos^2 A - 1} = \sin A$ {double angle formula}

$\therefore\ \dfrac{2\cos A}{2\cos^2 A - 1} = 1$ $\{\sin A \neq 0\}$

$2\cos A = 2\cos^2 A - 1$

$\therefore\ 2\cos^2 A - 2\cos A - 1 = 0$

b $\cos A = \dfrac{2 \pm \sqrt{(-2)^2 - 4\times 2\times(-1)}}{2\times 2}$

$= \dfrac{1\pm\sqrt{3}}{2}$

But $-1 \leqslant \cos A \leqslant 1$, so $\cos A = \dfrac{1-\sqrt{3}}{2}$

CALCULATORS

1 **a** Length of arc $= \frac{\theta}{360} \times 2\pi r$

$= \frac{40}{360} \times 2\pi \times 8$

≈ 5.59 cm

So, perimeter $\approx 8 + 8 + 5.59$

≈ 21.6 cm

b Area of sector $= \frac{\theta}{360} \times \pi r^2$

$= \frac{40}{360} \times \pi \times 8^2$

$\approx 22.3\ \text{cm}^2$

2 **a** Length of arc $= \theta r$

$= 2.4 \times 12$

$= 28.8$ cm

b Area of sector $= \frac{1}{2}\theta r^2$

$= \frac{1}{2} \times 2.4 \times 12^2$

$= 172.8\ \text{cm}^2$

Area of circle $= \pi r^2$

$= \pi \times 12^2$

$= 144\pi\ \text{cm}^2$

So, shaded area $= 144\pi - 172.8$

$\approx 280\ \text{cm}^2$

3 **a** Area $= \frac{1}{2}ab\sin\theta$

$= \frac{1}{2} \times 4 \times 4 \times \sin\theta$

$\therefore\ 4 = 8\sin\theta$

$\therefore\ \sin\theta = \frac{1}{2}$

$\therefore\ \theta = \frac{5\pi}{6}$ {since θ is obtuse}

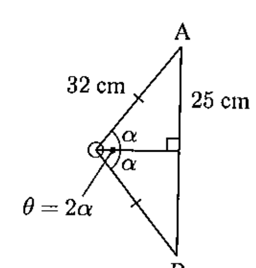
b Using the cosine rule,

$x^2 = 4^2 + 4^2 - 2\times 4\times 4\times\cos(\frac{5\pi}{6})$

$\therefore\ x = \sqrt{4^2 + 4^2 - 2\times 4\times 4\times\cos(\frac{5\pi}{6})}$

$\therefore\ x \approx 7.73$

4 **a**



$\sin\alpha = \frac{25}{32}$

$\therefore\ \alpha = \sin^{-1}\left(\frac{25}{32}\right)$

$\theta = 2\alpha$

$= 2\sin^{-1}(\frac{25}{32})$

≈ 1.793 radians

b Area of segment

= area of sector − area of triangle

$= \frac{1}{2}\theta r^2 - \frac{1}{2}r^2\sin\theta$ {where θ is in radians}

$= \frac{1}{2} \times 1.793 \times 32^2 - \frac{1}{2} \times 32^2 \times \sin(1.793)$

$\approx 419\ \text{cm}^2$

5 **a** Area $= \frac{1}{2} \times 5.4 \times 7.8 \times \sin 125° \approx 17.3\ \text{cm}^2$

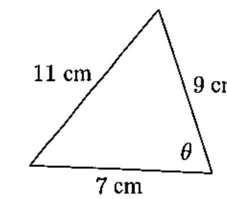
b Using the cosine rule,

$AC^2 = 5.4^2 + 7.8^2 - 2\times 5.4\times 7.8\times\cos 125°$

$\therefore\ AC = \sqrt{5.4^2 + 7.8^2 - 2\times 5.4\times 7.8\times\cos 125°}$

$\therefore\ AC \approx 11.8$ cm

6 **a**



The largest angle is opposite the longest side.

$\cos\theta = \dfrac{9^2 + 7^2 - 11^2}{2\times 9\times 7}$ {cosine rule}

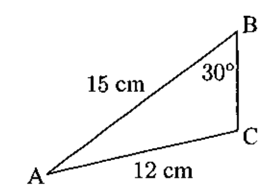
$\therefore\ \cos\theta = \frac{9}{126}$

$\therefore\ \theta \approx 85.9°$

b Area of triangle $\approx \frac{1}{2} \times 7 \times 9 \times \sin 85.9°$

$\approx 31.4\ \text{cm}^2$

7 **a**



$\dfrac{\sin C}{15} = \dfrac{\sin 30°}{12}$ {sine rule}

$\therefore\ C = \sin^{-1}\left(\dfrac{15\sin 30°}{12}\right)$

$\therefore\ C \approx 38.7°$ or $141.3°$

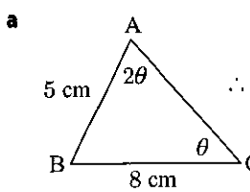
b From **a**, $A\hat{C}B \approx 38.7°$ or $141.3°$

$\therefore\ B\hat{A}C \approx 180° - 30° - 38.7°$ or $180° - 30° - 141.3°$

$\therefore\ B\hat{A}C \approx 111°$ or $8.68°$

$\therefore\ B\hat{A}C \approx 8.68°$ {since $B\hat{A}C$ is acute}

8 **a**



$\dfrac{\sin 2\theta}{8} = \dfrac{\sin\theta}{5}$ {sine rule}

$\therefore\ \dfrac{2\sin\theta\cos\theta}{8} = \dfrac{\sin\theta}{5}$

$\therefore\ \cos\theta = \frac{4}{5}$

b $A\hat{B}C = \pi - 3\theta$

$= \pi - 3\cos^{-1}(\frac{4}{5})$

≈ 1.211

Area of triangle

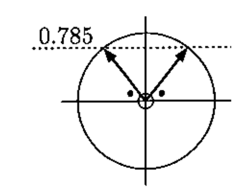
$\approx \frac{1}{2} \times 5 \times 8 \times \sin(1.211)$

$\approx 18.7\ \text{cm}^2$

9 **a** $\sin x = 0.785$

$\therefore\ x \approx 0.9027$ or $\pi - 0.9027$

$\therefore\ x \approx 0.903$ or 2.24



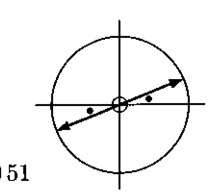
b $2\cos x = 5\sin x$

$\therefore\ \frac{2}{5} = \dfrac{\sin x}{\cos x}$

$\therefore\ \tan x = \frac{2}{5}$

$\therefore\ x \approx 0.380\,51$ or $\pi + 0.380\,51$

$\therefore\ x \approx 0.381$ or 3.52



c $\tan 3x = 0.9$

Since $0 \leqslant x \leqslant 2\pi$,

$0 \leqslant 3x \leqslant 6\pi$

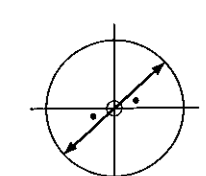
$\therefore\ 3x \approx 0.733,\ 0.733 + \pi,$

$0.733 + 2\pi,\ 0.733 + 3\pi,$

$0.733 + 4\pi,\ 0.733 + 5\pi$

$\therefore\ x \approx 0.244,\ 1.29,\ 2.34,$

$3.39,\ 4.43,\ 5.48$



d $4\sin^2 x = \cos^2 x$

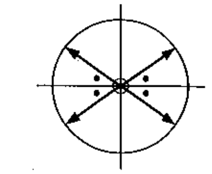
$\therefore\ \dfrac{\sin^2 x}{\cos^2 x} = \frac{1}{4}$

$\therefore\ \tan^2 x = \frac{1}{4}$

$\therefore\ \tan x = \pm\frac{1}{2}$

$\therefore\ x \approx 0.464,\ 2.68,$

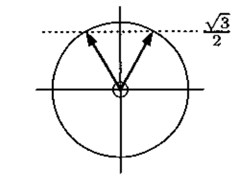
$3.61,\ 5.82$



10 **a** $2\sin\theta = \sqrt{3}$, $-2\pi \leqslant \theta \leqslant 2\pi$

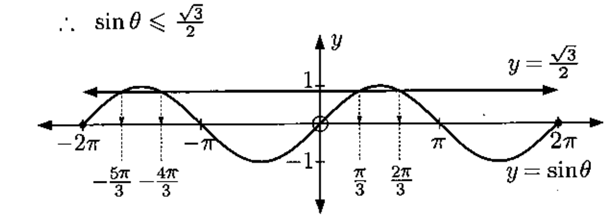
$\therefore\ \sin\theta = \frac{\sqrt{3}}{2}$

$\therefore\ \theta = -\frac{5\pi}{3}, -\frac{4\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$



b $2\sin\theta \leqslant \sqrt{3}$, $-2\pi \leqslant \theta \leqslant 2\pi$

$\therefore\ \sin\theta \leqslant \frac{\sqrt{3}}{2}$

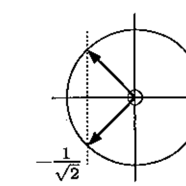


$\therefore\ -2\pi \leqslant \theta \leqslant -\frac{5\pi}{3},\ -\frac{4\pi}{3} \leqslant \theta \leqslant \frac{\pi}{3}$, or $\frac{2\pi}{3} \leqslant \theta \leqslant 2\pi$

11 **a** $\sqrt{2}\cos\theta = -1$, $-2\pi \leqslant \theta \leqslant 2\pi$

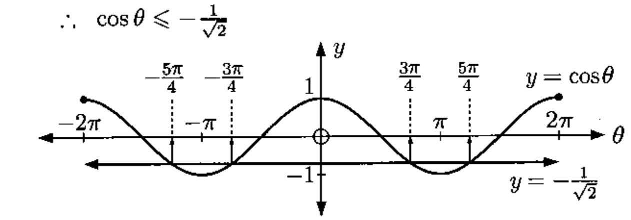
$\therefore\ \cos\theta = -\frac{1}{\sqrt{2}}$

$\therefore\ \theta = -\frac{5\pi}{4}, -\frac{3\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$



b $\sqrt{2}\cos\theta \leqslant -1$, $-2\pi \leqslant \theta \leqslant 2\pi$

$\therefore\ \cos\theta \leqslant -\frac{1}{\sqrt{2}}$



$\therefore\ -\frac{5\pi}{4} \leqslant \theta \leqslant -\frac{3\pi}{4}$ or $\frac{3\pi}{4} \leqslant \theta \leqslant \frac{5\pi}{4}$

12 $d = a + b\sin\left(\dfrac{2\pi t}{k}\right)$

a The period $= \dfrac{2\pi}{\frac{2\pi}{k}} = 12$, so $k = 12$

b When $t = 3$ and 15, $d_{\max} = 12.5$

At this time, $\sin\left(\dfrac{2\pi t}{k}\right) = 1$

$\therefore\ 12.5 = a + b$ .... (1)

When $t = 9$ and 21, $d_{\min} = 8.7$

At this time, $\sin\left(\dfrac{2\pi t}{k}\right) = -1$

$\therefore\ 8.7 = a - b$ .... (2)

Adding (1) and (2), we get $2a = 21.2$

$\therefore\ a = 10.6$ and $b = 1.9$

Check: $d = 10.6 + 1.9\sin\left(\dfrac{\pi t}{6}\right)$ has max. value

$10.6 + 1.9 = 12.5$ when $\sin\left(\dfrac{\pi t}{6}\right) = 1$

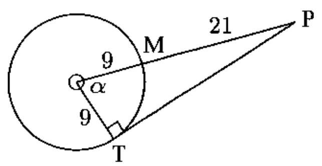
$$\therefore \quad \frac{\pi t}{6} = \frac{\pi}{2} + c2\pi$$
$$\therefore \quad \frac{t}{6} = \frac{1}{2} + 2c$$
$$\therefore \quad t = 3 + 12c \quad \checkmark$$

c For the depth of water to be 10 m,

$$10 = 10.6 + 1.9\sin\left(\frac{2\pi t}{12}\right)$$
$$\therefore \quad t \approx 6.6136,\ 11.386,\ 18.614,\ 23.386$$

So, the first time after 9 pm that the depth of water is 10 m, is at 23.386 hours after midnight, or 11:23 pm.

13



a In $\triangle$OTP, $\cos\alpha = \frac{9}{30} = 0.3$

$$\therefore \quad \alpha = \cos^{-1}(0.3)$$
$$\therefore \quad \alpha \approx 72.542^\circ$$
$$\therefore \quad \alpha \approx 72.5^\circ$$

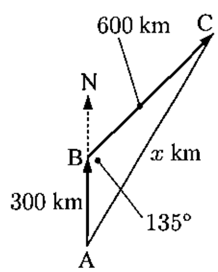
b

$$\text{Area of } \triangle\text{OTP} \approx \tfrac{1}{2} \times 9 \times 30 \times \sin 72.542^\circ \approx 128.78 \text{ cm}^2$$

$$\text{Area of sector OTM} = \left(\frac{\alpha}{360}\right) \times \pi \times 9^2 \approx \frac{72.542}{360} \times \pi \times 81 \approx 51.28 \text{ cm}^2$$

$$\therefore \quad \text{area of shaded region} \approx 128.78 - 51.28 \approx 77.5 \text{ cm}^2$$

14 **a**



$\text{AB} = 400 \times \frac{3}{4} = 300$ km

$\text{BC} = 400 \times 1\frac{1}{2} = 600$ km

Using the cosine rule,

$$x^2 = 300^2 + 600^2 - 2 \times 300 \times 600 \cos 135^\circ$$
$$\therefore \quad x = \sqrt{300^2 + 600^2 - 600^2 \times \cos 135^\circ}$$
$$\therefore \quad x \approx 839.38$$

$\therefore$ the distance is about 839 km.

b

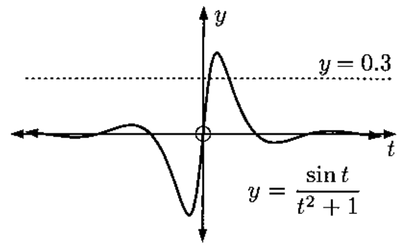
$$\text{time} = \frac{\text{distance}}{\text{speed}} \approx \frac{839.38}{400} \approx 2.098 \approx 2 \text{ hours } 6 \text{ minutes}$$

15 **a** We need to solve

$$I = \frac{\sin t}{t^2+1} = 0, \quad -4\pi \leqslant t \leqslant 4\pi$$
$$\therefore \quad \sin t = 0$$
$$\therefore \quad t = 0, \pm\pi, \pm 2\pi, \pm 3\pi, \pm 4\pi \text{ milliseconds}$$

There is zero electrical impulse when $t = 0, \pm\pi, \pm 2\pi, \pm 3\pi, \pm 4\pi$ milliseconds.

b Consider the graphs of $y = \dfrac{\sin t}{t^2+1}$ and $y = 0.3$.



Using technology, the t-coordinates of the points of intersection are about 0.342 and 1.526.

$\therefore$ the solutions of $\dfrac{\sin t}{t^2+1} = 0.3$

are $t \approx 0.342$ and $t \approx 1.526$

c The two solutions in **b** represent the times at which the electrical impulse is 0.3 units. Between these times, the electrical impulse reaches its maximum.

16 **a**

$$\tan\theta = 2\cos\theta$$
$$\therefore \quad \frac{\sin\theta}{\cos\theta} = 2\cos\theta$$
$$\therefore \quad \sin\theta = 2\cos^2\theta \qquad \{\text{as } 0 < \theta < \tfrac{\pi}{2},\ \cos\theta \neq 0\}$$
$$\therefore \quad \sin\theta = 2(1 - \sin^2\theta)$$
$$\therefore \quad \sin\theta = 2 - 2\sin^2\theta$$
$$\therefore \quad 2\sin^2\theta + \sin\theta - 2 = 0$$

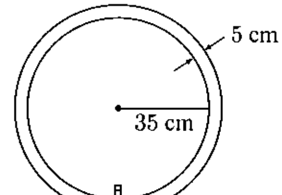
b

$$\sin\theta = \frac{-1 \pm \sqrt{1^2 - 4 \times 2(-2)}}{2 \times 2} = \frac{-1 \pm \sqrt{17}}{4}$$

But $-1 \leqslant \sin\theta \leqslant 1$,

$$\text{so} \quad \sin\theta = \frac{-1 + \sqrt{17}}{4} \approx 0.7808$$
$$\therefore \quad \theta \approx 0.896 \text{ or } 2.25$$
$$\therefore \quad \theta \approx 0.896 \quad \{\text{as } 0 < \theta < \tfrac{\pi}{2}\}$$

17



a **i** At 0 seconds, the valve is at its lowest position, closest to the road, so the height of the valve above the road is 5 cm.

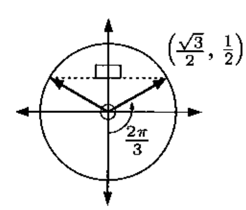
ii The wheel rotates at a constant speed of 4 revolutions per second.

$\therefore$ after $\frac{1}{12}$ second, the wheel has rotated

$\frac{1}{12} \times 4 = \frac{1}{3}$ revolution.

$\therefore$ the valve will have moved through an angle of $\frac{2\pi}{3}$.

So, the valve will be at a height of $1\frac{1}{2}$ times the radius of the wheel, plus 5 cm.



$\therefore$ the height of the valve above the road

$= 1.5 \times 35 + 5$

$= 57.5$ cm