Midterm Review: Logs, Exp, Sequences, & Series

① $u_1 = 2$
$u_5 = 32$

Method 1

$32 = 2 + (5-1)d$

$30 = 4d$

$d = \frac{30}{4} = \frac{15}{2} = 7.5$

$S_5 = \frac{5}{2}\left(2(2) + (5-1)(7.5)\right)$

$S_5 = 85$

Method 2

$S_5 = \frac{5}{2}(2 + 32)$

$S_5 = 85$

② The series is arithmetic w/ $d = 3$.

$3750 = 3 + (n-1)(3)$

$3750 = 3 + 3n - 3$

$3750 = 3n$

$1250 = n$

$S_{1250} = \frac{1250}{2}(3 + 3750)$

$= 2{,}345{,}625$

③ $r = -\frac{2}{3}$

$S_\infty = \dfrac{\frac{2}{3}}{1-\left(-\frac{2}{3}\right)} = \dfrac{\frac{2}{3}}{\frac{5}{3}} = \frac{2}{5}$ or $.4$

④ Ashley is arithmetic w/ $d = 2$ & $u_1 = 12$.

a) $S_{15} = \frac{15}{2}\left(2(12) + (15-1)(2)\right) = 390$

Billie is geometric w/ $r = 1.1$ & $u_1 = 12$.

b) i. $u_3 = 12(1.1)^{3-1} = 14.52$

ii. $S_{15} = \dfrac{12(1.1^{15} - 1)}{1.1 - 1} = 381$ (to 3 sig figs)

c) $50 = 12(1.1)^{n-1}$

$\frac{50}{12} = (1.1)^{n-1}$

$\log_{1.1}\left(\frac{50}{12}\right) = n - 1$

$n = 15.97$, so week 16

⑤ $9^{x-1} = \left(\frac{1}{3}\right)^{2x}$

Method 1

$$\left(3^2\right)^{x-1} = \left(3^{-1}\right)^{2x}$$
$$3^{2x-2} = 3^{-2x}$$
$$2x-2 = -2x$$
$$4x = 2$$
$$x = \frac{1}{2}$$

Method 2

$$\log_9 \left(\frac{1}{3}\right)^{2x} = x-1$$
$$2x\left(\log_9 \frac{1}{3}\right) = x-1$$
$$2x\left(-\frac{1}{2}\right) = x-1$$
$$-x = x-1$$
$$-2x = -1$$
$$x = \frac{1}{2}$$

⑥ a) $\log_5 x^2 = 2\log_5 x$
$= 2(y) = 2y$

b) $\log_5\left(\frac{1}{x}\right) = \log_5 1 - \log_5 x$
$= 0 - y = -y$

c) $\log_{25} x = \dfrac{\log_5 x}{\log_5 25} = \dfrac{y}{2}$

* Need to change base *

⑦ Growth formula: $y = P(1+r)^t$ pick any value for P

$$100 = 50(1+.023)^t$$
$$2 = 1.023^t$$
$$\log_{1.023} 2 = t$$
$$30 = t$$
(nearest minute)

(8) $n = 800e^{0.13t}$

a) $n = 800e^{0} = 800$

b) omit

c) change to "After K minutes, the number of bacteria is greater than 10,000..."

$$10{,}000 = 800e^{0.13t}$$
$$12.5 = e^{0.13t}$$
$$\log_e 12.5 = 0.13t$$
$$19.4 = t$$

(9) * Have to use log properties to split up first *

$$\log_{10}\left(\frac{P}{QR^3}\right)^2 = 2\log_{10}\left(\frac{P}{QR^3}\right)$$
$$= 2\left[\log_{10} P - \log_{10} QR^3\right]$$
$$= 2\left[\log_{10} P - (\log_{10} Q + \log_{10} R^3)\right]$$
$$= 2\left[\log_{10} P - (\log_{10} Q + 3\log_{10} R)\right]$$
$$= 2\left[x - (y + 3z)\right]$$
$$= 2\left[x - y - 3z\right] = \boxed{2x - 2y - 6z}$$

(10) $\log_{27} x = 1 - \log_{27}(x - .4)$

$$\log_{27} x = \log_{27} 27 - \log_{27}(x - .4)$$
$$\log_{27} x = \log_{27} \frac{27}{x - .4}$$

$$\frac{x}{1} = \frac{27}{x - .4}$$
$$27 = x^2 - .4x \rightarrow \text{solve w/ quad. formula or graph it}$$
$$x = \cancel{-5}, 5.4$$