Differential Equations Optional Assignment

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Problem Statement

1. Find the general solution of the differential equation $\frac{dy}{dx} = y \sin x$.
2. Find the particular solution of the differential equation $\frac{dy}{dx} = y \sin x$ that satisfies the following initial condition:
   a. $\left(0, \frac{2}{e}\right)$
   b. $\left(0, \frac{1}{e}\right)$
   c. $\left(0, \frac{-2}{e}\right)$
3. Sketch the slope field for $\frac{dy}{dx} = y \sin x$, along with the three solutions found in part (2).

Solution

1. The first step in solving the differential equation $\frac{dy}{dx} = y \sin x$ is to separate the variables.

$$\frac{dy}{dx} = y \sin x$$

$$\frac{dy}{y} = \sin x \, dx$$

Next, integrate both sides of the equation with respect to the appropriate variables.

$$\int \frac{dy}{y} = \int \sin x \, dx$$

$$\ln|y| = -\cos x + C$$

Finally, we solve the equation for y in terms of x.

$$e^{\ln|y|} = e^{-\cos x + C}$$

$$y = e^{-\cos x} \cdot e^{C}$$

$$y = Ce^{-\cos x}$$

2. To find the particular solution, substitute the initial conditions into the general solution and solve for the constant C.

(a) For $\left(0, \frac{2}{e}\right)$

$$\frac{2}{e} = Ce^{-\cos 0}$$

$$\frac{2}{e} = Ce^{-1}$$

$$\frac{2}{e} = \frac{C}{e}$$

$$2 = C$$

$$\therefore y = 2e^{-\cos x}$$

(b) For $\left(0, \frac{1}{e}\right)$

$$\frac{1}{e} = Ce^{-\cos 0}$$

$$\frac{1}{e} = Ce^{-1}$$

$$\frac{1}{e} = \frac{C}{e}$$

$$1 = C$$

$$\therefore y = e^{-\cos x}$$

(c) For $\left(0, \frac{-2}{e}\right)$

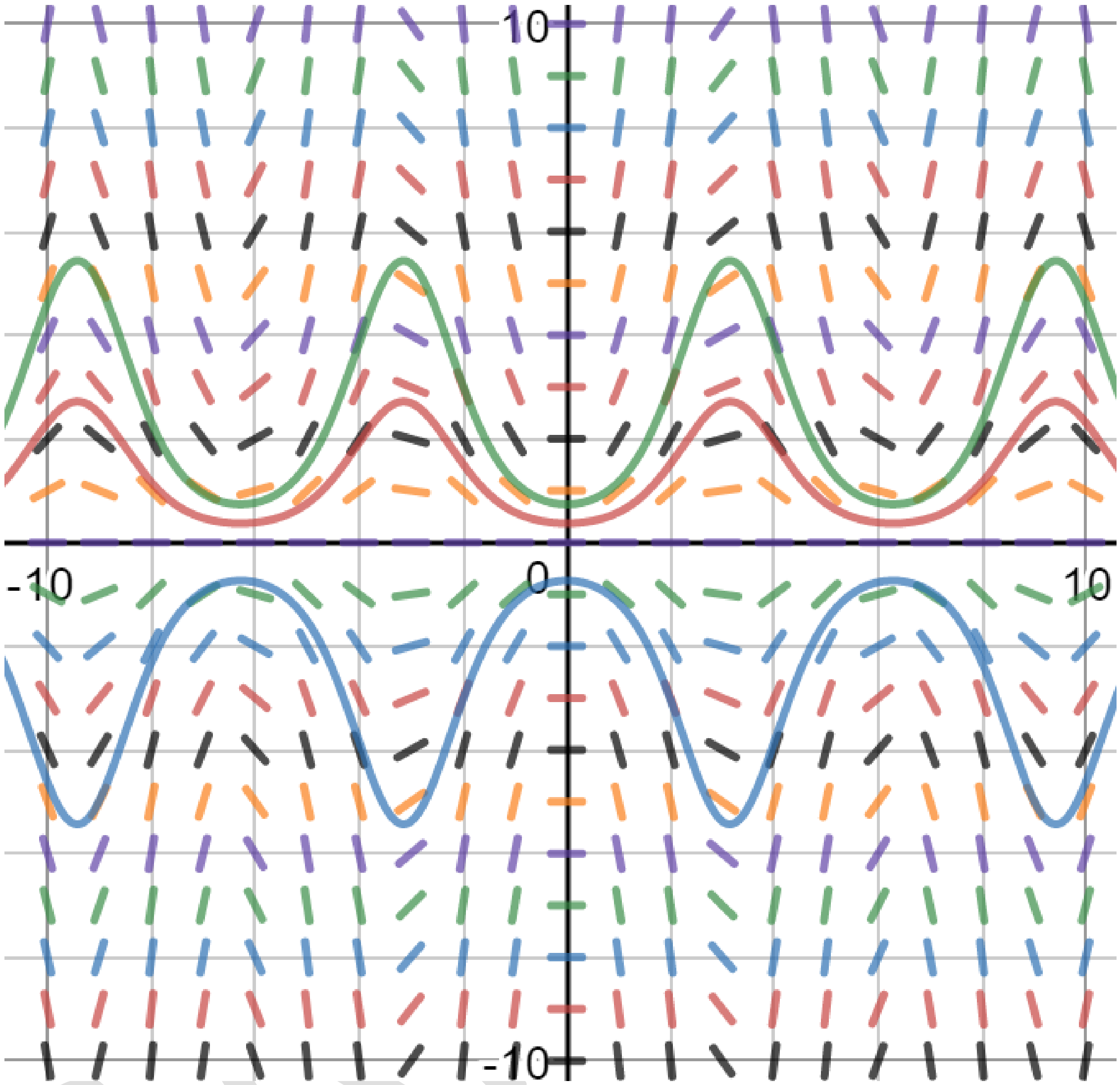
$$\frac{-2}{e} = Ce^{-\cos 0}$$

$$\frac{-2}{e} = Ce^{-1}$$

$$\frac{-2}{e} = \frac{C}{e}$$

$$-2 = C$$

$$\therefore y = -2e^{-\cos x}$$



3. The slope field for $\frac{dy}{dx} = y \sin x$ is shown below, along with the three solutions found in part (2).