Key

We will go over questions next class. Please do as much as you can, so we know what to continue working on next class. Use your packets and notes!!

1. The following frequency distribution of marks has ~~mean 4.5.~~

Mark	1	2	3	4	5	6	7
Frequency	2	4	6	9	x	9	4

(a) Find the value of x.

$$\frac{1(2)+2(4)+3(6)+4(9)+5x+6(9)+7(4)}{2+4+6+9+x+9+4} = 4.5$$

(4)

(b) Write down the standard deviation.

$$\frac{146+5x}{34+x} = 4.5$$

(2)

(Total 6 marks)

$$x = 14$$

$\sigma = 1.54$

2. The following table gives the examination grades for ~~120 students~~

Grade	Number of students	Cumulative frequency
1	9	9
2	25	34
3	35	p
4	q	109
5	11	120

$34 + 35 = p$
$p = 69$

(a) Find the value of p & q.

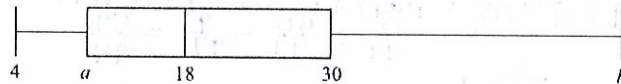$9 + 25 + 35 + q + 11 = 120$
$q = 40$

(4)

(b) Find the mean grade. $\bar{x} = 3.16$

(2)

(c) Write down the standard deviation.

$\sigma = 1.09$

(1)

(Total 7 marks)

3. The following diagram is a box and whisker plot for a set of data.

| 4 | a | 18 | 30 | b |

The interquartile range is 20 and the range is 40.

(a) Write down the median value. 18

(1)

(b) Find the value of a & b.

$30 - a = 20$ $b - 4 = 40$
$a = 10$ $b = 44$

(4)

(Total 5 marks)

4. A standard die is rolled 36 times. The results are shown in the following table.

Score	1	2	3	4	5	6
Frequency	3	5	4	6	10	8

(a) Write down the standard deviation. $\sigma = 1.61$

(2)

(b) Write down the median score. 4.5

(1)

(c) Find the interquartile range. $5 - 3 = 2$
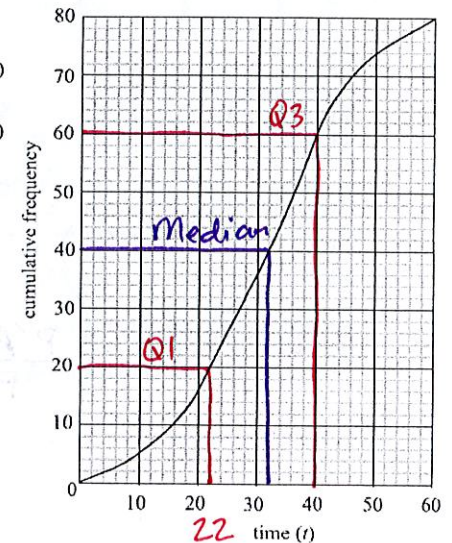
(3)

(Total 6 marks)

5. The following is a cumulative frequency diagram for the time t, in minutes, taken by ~~80~~ students to complete a task.

(a) Write down the median. 32

(1)
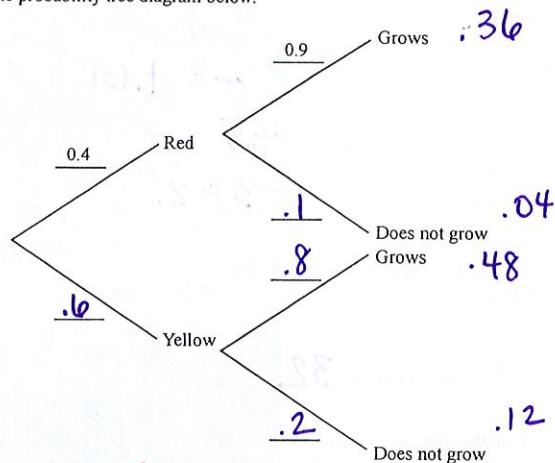
(b) Find the interquartile range.

$40 - 22 = 18$

(3)

(c) Complete the frequency table below.

Time (minutes)	Number of students
$0 \le t < 10$	5
$10 \le t < 20$	11
$20 \le t < 30$	20
$30 \le t < 40$	24
$40 \le t < 50$	14
$50 \le t < 60$	6

6. A packet of seeds contains 40% red seeds and 60% yellow seeds. The probability that a red seed grows is 0.9, and that a yellow seed grows is 0.8. A seed is chosen at random from the packet.

(a) Complete the probability tree diagram below.

0.9 — Grows $.36$

0.4 — Red

$.1$ — Does not grow $.04$

$.8$ — Grows $.48$

$.6$ — Yellow

$.2$ — Does not grow $.12$

(3)

(b) (i) Calculate the probability that the chosen seed is red and grows. 0.36

(ii) Calculate the probability that the chosen seed grows. $.36 + .48 = .84$

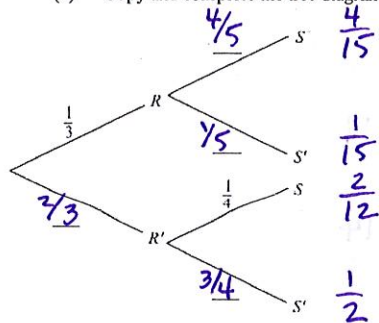(iii) Given that the seed grows, calculate the probability that it is red.

$$P(R \mid G) = \frac{P(R \cap G)}{P(G)} = \frac{.36}{.84} = .43 \text{ or } \frac{3}{7}$$

(7)
(Total 10 marks)

7. The following probabilities were found for two events R and S.

$$P(R) = \frac{1}{3}, \quad P(S \mid R) = \frac{4}{5}, \quad P(S \mid R') = \frac{1}{4}.$$

(a) Copy and complete the tree diagram.

$4/5$ — S $\frac{4}{15}$

$\frac{1}{3}$ — R

$1/5$ — S' $\frac{1}{15}$

$\frac{1}{4}$ — S $\frac{2}{12}$

$2/3$ — R'

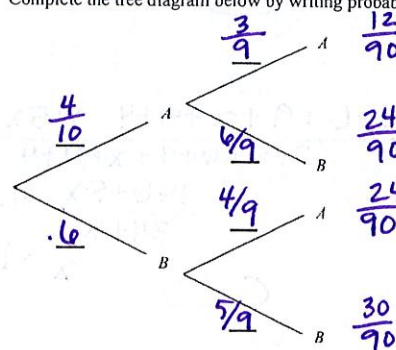$3/4$ — S' $\frac{1}{2}$

(3)

(b) Find the following probabilities.

(i) $P(R \cap S)$. $\frac{1}{3} \cdot \frac{4}{5} = \frac{4}{15}$

(ii) $P(S)$. $\frac{4}{15} + \frac{2}{12} = \frac{13}{30}$

(iii) $P(R \mid S)$.

$$= \frac{P(R \cap S)}{P(S)} = \frac{\frac{4}{15}}{\frac{13}{30}} = \frac{4}{15} \cdot \frac{30}{13} = \frac{8}{13}$$

8. A bag contains four apples (A) and six bananas (B). A fruit is taken from the bag and eaten. Then a second fruit is taken and eaten.

(a) Complete the tree diagram below by writing probabilities in the spaces provided.

$\frac{3}{9}$ — A $\frac{12}{90}$

$\frac{4}{10}$ — A

$6/9$ — B $\frac{24}{90}$

$4/9$ — A $\frac{24}{90}$

$.6$ — B

$5/9$ — B $\frac{30}{90}$

(3)

(b) Find the probability that one of each type of fruit was eaten.

$$P(AB \text{ or } BA) = \frac{24}{90} + \frac{24}{90} = \frac{48}{90} = \frac{8}{15}$$

(3)
(Total 6 marks)

9. The eye colour of 97 students is recorded in the chart below.

	Brown	Blue	Green	Total
Male	21	16	9	46
Female	19	19	13	51

One student is selected at random.

(a) Write down the probability that the student is a male. $\frac{46}{97}$

(b) Write down the probability that the student has green eyes, given that the student is a female.

$$P(G \mid F) = \frac{P(G \cap F)}{P(F)} = \frac{\frac{13}{97}}{\frac{51}{97}} = \frac{13}{51}$$

(c) Find the probability that the student has green eyes or is male.

$$P(G \cup M) = P(G) + P(M) - P(G \cap M)$$
$$= \frac{22}{97} + \frac{46}{97} - \frac{9}{97} = \frac{59}{97}$$

(Total 6 marks)

10. Let A and B be independent events, where $P(A) = 0.6$ and $P(B) = x$.

(a) Write down an expression for $P(A \cap B)$. $= P(A) \cdot P(B)$
$$= .6x$$

(1)

(b) Given that $P(A \cup B) = 0.8$, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$$.8 = .6 + x - .6x$$

(i) find x; $.2 = .4x$
$$.5 = x$$

(ii) find $P(A \cap B)$. $(.6)(.5) = .3$

(4)

(c) Hence, explain why A and B are not mutually exclusive.

To be mutually exclusive, $P(A \cap B) = 0$, but above we see that $P(A \cap B) = .3$

(1)

4

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$.52 = P(A) + 2P(A) - [P(A) \cdot P(B)]$
$.52 = 3P(A) - [P(A) \cdot 2P(A)]$
$.52 = 3P(A) - 2[P(A)]^2 \longrightarrow P(A) = .2$
(Total 6 marks) so $P(B) = .4$

11. Consider the <u>independent</u> events A and B. Given that $P(B) = 2P(A)$, and $P(A \cup B) = 0.52$, find $P(B)$.

12. In any given season, a soccer team plays 65 % of their games at home.
When the team plays at home, they win 83 % of their games.
When they play away from home, they win 26 % of their games.

The team plays one game.

$.65$ H $\overset{.83}{\underset{.17}{\big\langle}}$ $\overset{W \ .5395}{L \ .1105}$ **(Total 7 marks)**

$.35$ A $\overset{.26}{\underset{.74}{\big\langle}}$ $\overset{W \ .091}{L \ .259}$

(4)

(a) Find the probability that the team wins the game.
$.5395 + .091 = .6305$

(b) If the team does not win the game, find the probability that the game was played at home.
$P(H|L) = \dfrac{P(H \cap L)}{P(L)} = \dfrac{.1105}{.3695} = .299$ (4)

(Total 8 marks)

13. The number of hours of sleep of 21 students are shown in the frequency table below.

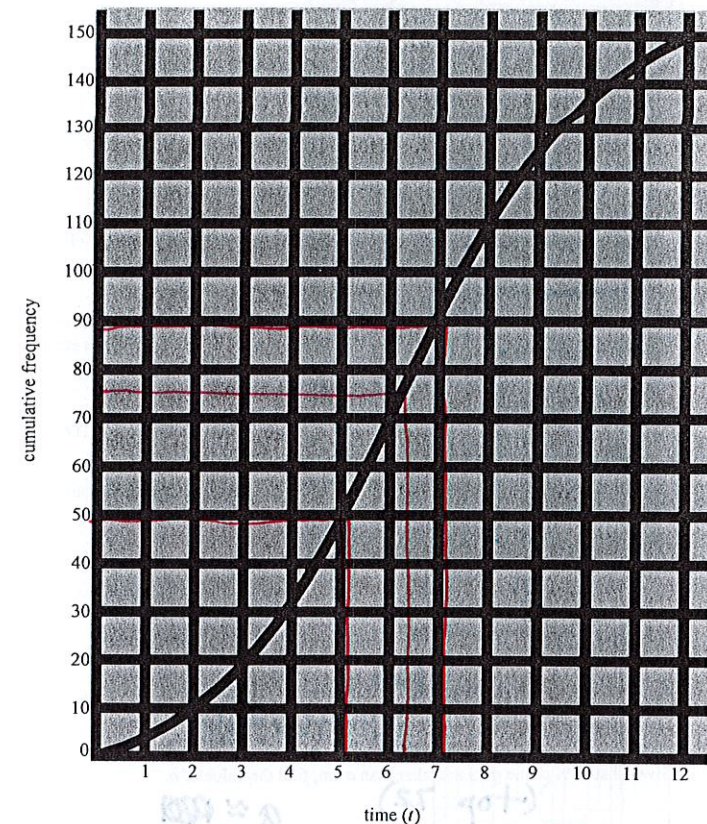| Hours of sleep	Number of students
4	2
5	5
6	4
7	3
8	4
10	2
12	1

Find

(a) the median; 6

(b) the lower quartile; 5

(c) the interquartile range. $8 - 5 = \boxed{3}$

(Total 6 marks)

14. The following is the cumulative frequency curve for the time, t minutes, spent by 150 people in a store on a particular day.

(a) (i) How many people spent less than 5 minutes in the store? 50

(ii) Find the number of people who spent between 5 and 7 minutes in the store. $90 - 50 = 40$

(iii) Find the median time spent in the store. ~ 6.25 min
(6)

(b) Given that 40% of the people spent longer than k minutes, find the value of k. 7 minutes
(+top 60)
(3)

(c) (i) On your answer sheet, copy and complete the following frequency table.

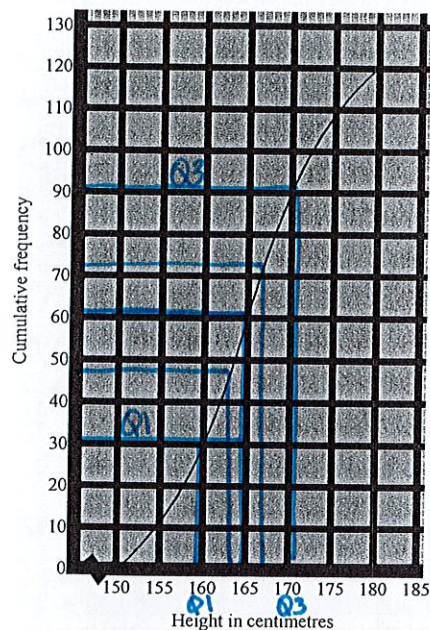| t (minutes)	$0 \le t < 2$	$2 \le t < 4$	$4 \le t < 6$	$6 \le t < 8$	$8 \le t < 10$	$10 \le t < 12$
Frequency	10	23	37	40	25	15

(ii) Hence, calculate an estimate for the mean time spent in the store.
hint: use midinterval values
(5)
(Total 14 marks)
$\bar{x} \approx 6.23$

15. The cumulative frequency graph below shows the heights of 120 girls in a school.

Height in centimetres

(a) Using the graph

(i) write down the median; **165**

(ii) find the interquartile range. **170 − 160 = 10**

(b) Given that 60% of the girls are taller than a cm, find the value of a.

(top 72) $a \approx$ ~~160~~ **164**

(Total 6 marks)

16. A student measured the diameters of 80 snail shells. His results are shown in the following cumulative frequency graph. The lower quartile (LQ) is 14 mm and is marked clearly on the graph.

LQ = 14
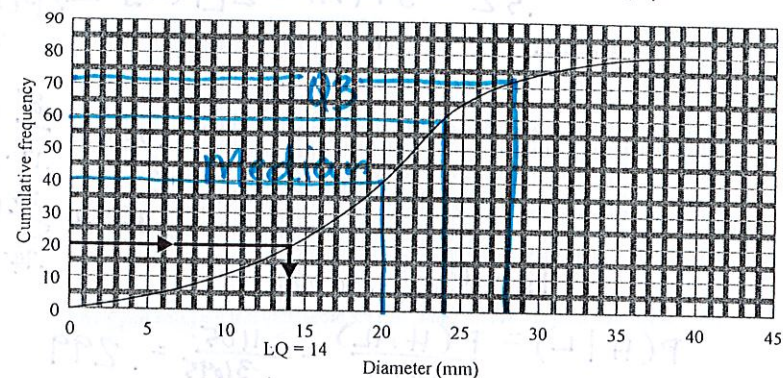
Diameter (mm)

(a) On the graph, mark clearly in the same way and write down the value of

(i) the median; **20**

(ii) the upper quartile. **24**

(b) Write down the interquartile range. **24 − 14 = 10**

(c) Find the smallest diameter of the upper 10% of the snails.

top 8   **28 mm**

(Total 8 marks)