Unit 3 Review

* start w/ chain rule, then use product rule *

① $h(x) = (f(x)\cdot g(x))^{1/2}$

$h'(x) = \frac{1}{2}(f(x)g(x))^{-1/2}[f(x)g'(x) + g(x)f'(x)]$

$h'(-3) = \frac{1}{2}(2\cdot 4)^{-1/2}[2(-2) + 4(\frac{1}{2})]$

$h'(-3) = \frac{1}{2}\cdot\frac{1}{\sqrt{8}}[-2] = \boxed{\frac{-1}{\sqrt{8}}}$

② Continuous = no holes, breaks, VA

Differentiable = no holes, breaks, cusps, VA

→ only at $x = 3$ is f continuous but not differentiable.

③ a. * make sure top & bottom pieces are equal at $x=2$.

$-3(2)^2 + 4(2) + 2 \stackrel{?}{=} -3(2) + 4$

$-2 \neq -2$ justification: $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$

b. * make sure derivatives of top & bottom pieces are equal at $x=2$.

$2ax + b \stackrel{?}{=} a$

$2(-3)(2) + 4 \stackrel{?}{=} -3$

$-8 \neq -3$

justification: $\lim_{x\to 2^-} f'(x) \neq \lim_{x\to 2^+} f'(x)$

c. $4a + 2b + 2 = 2a + b$ AND $4a + b = a$

$4a + 2(-3a) + 2 = 2a - 3a$ $\qquad 3a = -b$

$-2a + 2 = 2a - 3a$ $\qquad -3a = b$

$2 = a$

$b = -6$

④ a. $\frac{d}{dx}(\ln x) = \frac{1}{x}$

b. $\frac{d}{dx}\left[(2x-3)^{1/3}\right] = \frac{1}{3}(2x-3)^{-2/3}(2)$

$= \frac{2}{3(2x-3)^{2/3}}$

c. $\frac{d}{dx}(\sin x)$ at $\frac{\pi}{2}$

$=\cos x$, then plug in $\frac{\pi}{2}$.

$=\cos\frac{\pi}{2}$

$=0$

d. $\frac{d}{dx}(\tan 3x) = \sec^2(3x)\cdot 3$

$= 3\sec^2(3x)$

⑤ * There is a typo, it's supposed to say if $h(x) = \frac{f(x)}{g(x)}$!

$h(x) = \frac{f(x)}{g(x)}$

$h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

$h'(7) = \frac{(1)(14)-(0)(\frac{1}{7})}{1^2} = 14$

⑥ OMIT for now...

⑧ a. $y' = \frac{(2x-1)(2)-(2x+1)(2)}{(2x-1)^2}$

$y' = \frac{4x-2-4x-2}{(2x-1)^2}$

$y' = \frac{-4}{(2x-1)^2}$

b. $y = \cot(2t^{-1})$

$y' = -\csc^2(2t^{-1})\cdot -2t^{-2}$

$= \frac{2\csc^2(\frac{2}{t})}{t^2}$

) $y = x(2x+1)^{1/2}$

$y' = x \cdot \frac{1}{2}(2x+1)^{-1/2}(2) + (2x+1)^{1/2}(1)$

$y' = \frac{x}{(2x+1)^{1/2}} + (2x+1)^{1/2} = \frac{3x+1}{(2x+1)^{1/2}}$ (Hint: get a common denominator)

d. $r = (\tan(3-\theta^2))^2$

$\frac{dr}{d\theta} = 2(\tan(3-\theta^2)) \cdot \sec^2(3-\theta^2)(-2\theta)$

$\frac{dr}{d\theta} = -4\theta \tan(3-\theta^2)\sec^2(3-\theta^2)$

e. $y = \ln(x^{1/2})$

$\frac{dy}{dx} = \frac{1}{x^{1/2}} \cdot \frac{1}{2}x^{-1/2} = \frac{1}{2x}$

f. $y = (x)(e^{-x})$

$\frac{dy}{dx} = (x)(-e^{-x}) + (e^{-x})(1)$

$\frac{dy}{dx} = -xe^{-x} + e^{-x}$ or $e^{-x}(1-x)$