Review #2

1. $\frac{dy}{dx} = 12x^2 - 3$

2. $y = (\cos(x^3))^2$

$y' = 2(\cos(x^3))(-\sin(x^3))(3x^2)$

$= -6x^2 \cos x^3 \sin x^3$

3. $y(3) = \frac{9}{3} = 3$; $(3,3)$

$y' = \frac{(x^2-6)(3) - (3x)(2x)}{(x^2-6)^2}$

$y' = \frac{-3x^2-18}{(x^2-6)^2}$

$y'(3) = \frac{-27-18}{9} = -5$

$y - 3 = -5(x-3)$

4. $f'(x) = x \cdot \frac{1}{2}(2x-3)^{-1/2}(2) + (2x-3)^{1/2}(1)$

$= x(2x-3)^{-1/2} + (2x-3)^{1/2}$

$= (2x-3)^{-1/2}\left[x + (2x-3)\right]$

$= \frac{3x-3}{(2x-3)^{1/2}}$

5. $\left.\frac{dy}{dx}\right|_{(3,2)} = \frac{-2(2)^2}{4(3)(2)-3} = \frac{-8}{21}$ slope of normal $= \frac{21}{8}$

slope of tangent

6. $f'(x) = (x-1)\cdot 3(x^2+2)^2(2x) + (x^2+2)^3(1)$

$= 6x(x-1)(x^2+2)^2 + (x^2+2)^3$

$= (x^2+2)^2\left[6x(x-1) + x^2+2\right]$

$= (x^2+2)^2(7x^2-6x+2)$

7. $y' = \dfrac{(3x-1)(2) - (2x+5)(3)}{(3x-1)^2}$

$= \dfrac{-17}{(3x-1)^2}$

8. $y' = -\sin(2x)\cdot 2 = -2\sin(2x)$

$y'(\frac{\pi}{4}) = -2(1)$

$y(\frac{\pi}{4}) = 0$

$\rightarrow y - 0 = -2(x - \frac{\pi}{4})$

9. $f'(x) = \sec^2 x + \cos x$

10. $f'(x) = x^3 + x^2 - 2x$

$0 = x(x^2 + x - 2)$

$0 = x(x+2)(x-1)$

$x = 0, 1, -2$

11. $f'(x) = 12x^2 - 5$

$f'(-1) = 7$

$f(-1) = 4$

$\rightarrow y - 4 = 7(x+1)$

12. $f(x) = 4x^{-1}$

a. $f'(x) = -4x^{-2}$ or $\dfrac{-4}{x^2}$

b. $f'(-2) = \dfrac{-4}{4} = -1$

c. $y + 2 = -1(x+2)$

13. $h'(x) = 8(e^x+5)^7(e^x)$
$= 8e^x(e^x+5)^7$

14. $f'(x) = x^2+3x-10$
$0 = x^2+3x-10$
$0 = (x+5)(x-2)$ → horizontal tangents at $x = -5$ & 2

15. $y' = \dfrac{(1+\sin x)(-\sin x) - (\cos x)(\cos x)}{(1+\sin x)^2}$

$y' = \dfrac{-\sin x - \sin^2 x - \cos^2 x}{(1+\sin x)^2} = \dfrac{-\sin x - 1}{(1+\sin x)^2} = \dfrac{-1}{1+\sin x}$

16. $y+3 = \frac{2}{7}(x-2)$

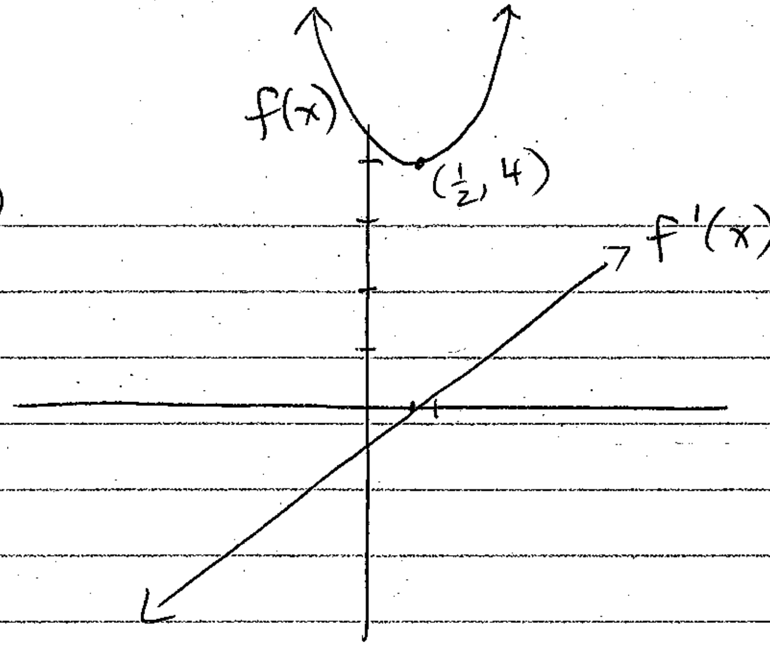
17. $f'(x) = g'(h(x))h'(x)$
$f'(2) = g'(h(2))h'(2) = g'(3)(4) = (5)(4) = 20$

18. $f'(x) = \dfrac{1}{(3x^2+x)} \cdot 6x+1 = \dfrac{6x+1}{3x^2+x}$

19. This means $f'(x) = -1$ (slope of tangent)
So $4x-5 = -1$
$4x = 4$
$\boxed{x=1}$

20. $(0,0)$ $(\pi, 1)$
ave. rate of change $= \dfrac{1-0}{\pi-0} = \dfrac{1}{\pi}$

21. vertex at $(\frac{1}{2}, 4)$
opens up



22. $f'(x)$ is below the x-axis anytime $f(x)$ has negative slope. $f'(x)$ is above the x-axis anytime $f(x)$ has positive slope.

23. Let $f(x) = 2x^2+1$ & $g(x) = x-3$.
$f'(x) = 4x$ $g'(x) = 1$

$(fg)' =$ product rule
$= (2x^2+1)(1) + (x-3)(4x)$
$= 6x^2 - 12x + 1$

not the same

$f' \cdot g' = (4x)(1)$