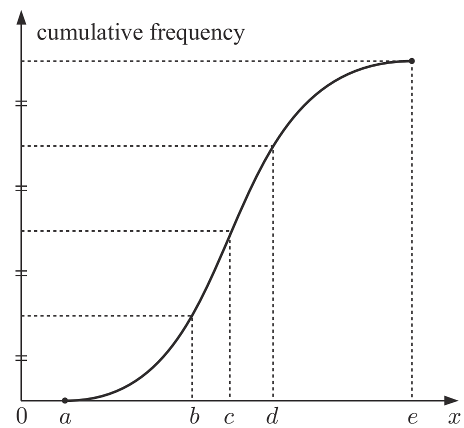
91 A straight line passes through A(2, 0, -3) and has direction vector $\mathbf{i} - \mathbf{j} + 2\mathbf{k}$.

- **a** Write down the vector equation of the line.
- **b** Write down parametric equations for the line.
- **c** What is represented by P$(2 + t, -t, -3 + 2t)$?
- **d** Find $\overrightarrow{BP}$ given the point B$(-1, 3, 5)$.
- **e** Find $\overrightarrow{BP} \bullet (\mathbf{i} - \mathbf{j} + 2\mathbf{k})$.
- **f** Hence find the value of t when [BP] is perpendicular to the original line.
- **g** What point on the original line is closest to B?

92 A cumulative frequency graph for the continuous random variable X is given alongside.



- **a** What is represented by:
  - **i** a **ii** b **iii** c
  - **iv** d **v** e?
- **b** What do these measure?
  - **i** $e - a$ **ii** $d - b$
- **c** Determine:
  - **i** $P(b < X \leqslant d)$ **ii** $P(X > b)$.
- **d** Draw an accurate boxplot for the data set.

B CALCULATOR QUESTIONS

EXERCISE 25B

1 Find n given that $\sum_{k=1}^{n} (2k - 31) = 0$.

2 Consider the function $f(x) = 5\ln(x - 4) + 2$.

- **a** Graph the function $y = f(x)$. Clearly label the axes intercepts and asymptotes.
- **b** Solve the equation $f(x) = 1$.
- **c** Graph the function $y = f^{-1}(x)$ on the same set of axes. Clearly label the axes intercepts and asymptotes.
- **d** Find the equation of the normal to the curve $y = f(x)$ at the point where $x = 5$.

3 Find the constant term in the expansion of $\left(x - \frac{1}{5x^2}\right)^9$.

4 The value of a cash investment after t years is given by $V = 7500 \times 2^{0.09t}$ dollars.

- **a** Find the initial value of the cash investment.
- **b** Find the value of the investment after:
  - **i** 5 years **ii** 15 years.
- **c** What was the percentage increase in the investment in the first five years?
- **d** How many years will it take for the investment to double in value?

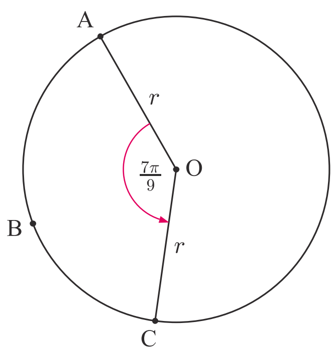
5 Consider the arithmetic sequence: $-900, -750, -600, -450, ...$

a Find the 20th term of the sequence. **b** Find the sum of its first 20 terms.

6 Functions f and g are defined by $f : x \longmapsto 2x$ and $g : x \longmapsto 1 - 5x^2$. Solve:

a $(f \circ g)(x) = -8$ **b** $(g \circ f)(x) = -8$ **c** $f'(x) = g'(x)$ **d** $f^{-1}(x) = g(x)$

7



The perimeter of sector OABC is 50π cm.

a Find r.

b Find the area of sector OABC.

c Calculate the side length of an equilateral triangle which has the same area as sector OABC.

8 Dora notices that the number of cups of coffee she drinks in a day varies depending on how much sleep she gets the previous night. She records the following data:

Time sleeping (hours)	6	8	6.5	5	9	5.5	7.5	6	8	8.5	7
Number of cups of coffee	3	1	2	4	0	4	2	4	0	1	2

a Draw a scatter diagram for this data.

b Calculate Pearson's product-moment correlation coefficient r for the data.

c Hence describe the correlation between these two variables.

9 **a** Draw a scatter diagram for the following data:

x	0	5	10	15	20	25	30	35	40	45	50	55	60
y	1	1.5	1.9	2	1.9	1.5	1	0.5	0.1	0	0.1	0.5	1

b Estimate:

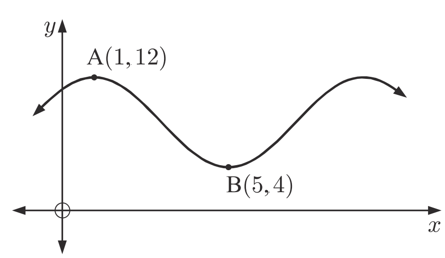
i the equation of the principal axis **ii** the maximum value

iii the period **iv** the amplitude.

c Hence find an equation of the form $y = a \sin bx + c$ that models the data.

10 The graph shown is a function of the form $f(x) = m\cos(n(x - p)) + r$.

It has a maximum turning point at A(1, 12) and a minimum turning point at B(5, 4).



a Determine the values of m, n, p, and r.

b Find $f(6)$.

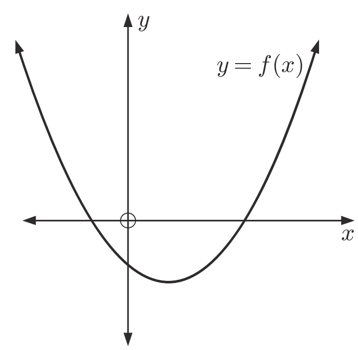
c Find the smallest positive value of x such that $f(x) = 10$.

11 The lines L_1 and L_2 have equations

$$\mathbf{r}_1 = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix} + t\begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} \quad \text{and} \quad \mathbf{r}_2 = \begin{pmatrix} -5 \\ 0 \\ 7 \end{pmatrix} + s\begin{pmatrix} 6 \\ -5 \\ 15 \end{pmatrix} \quad \text{respectively.}$$

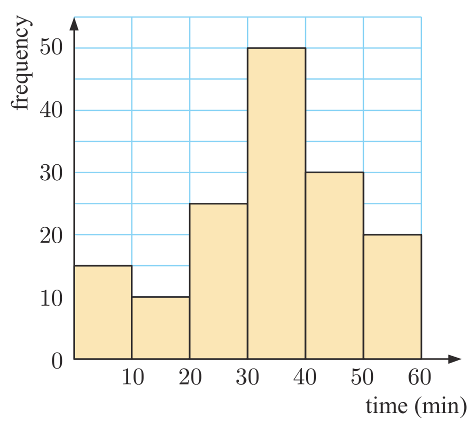
- **a** Determine the acute angle between L_1 and L_2.
- **b** Show that the point P$(-10, 3, -2)$ lies on L_1.
- **c** Does P lie on L_2? Give evidence for your answer.
- **d** Find the point of intersection of L_1 and L_2.
- **e** A third line L_3 has direction vector $\begin{pmatrix} a \\ 2 \\ 8 \end{pmatrix}$ and is perpendicular to L_1. Find a.

12 Consider the function $f(x) = (x+1)(x-\beta)$ where $\beta > 0$. A sketch of the function is shown alongside.



- **a** Determine the axes intercepts of the graph of $y = f(x)$.
- **b** Sketch the graphs of $f(x)$ and $g(x) = -f(x-1)$ on the same set of axes.
- **c** Hence, determine and label the axes intercepts of $y = g(x)$.

13



The frequency histogram illustrates the times taken by a group of people to solve a puzzle.

- **a** Construct a cumulative frequency graph for the data.
- **b** Hence estimate:
  - **i** the median time taken to solve the puzzle
  - **ii** the interquartile range of the data
  - **iii** the probability that a randomly selected person was able to complete the puzzle within 35 minutes.

14 A theatre has 30 rows of seats. There are 16 seats in the first row and each subsequent row has 2 more seats than the previous row. Seats are allocated randomly to all theatre patrons. Calculate the probability that a randomly chosen patron will be seated in the last row of the theatre.

15 Argentina's success rate at penalty shots is 86%. In a match against Brazil, Argentina takes 5 penalty shots.

- **a** Determine the probability that Argentina succeeds with all five of their penalty shots.
- **b** Max said that the probability of Argentina scoring *exactly* three of their 5 shots taken is $(0.86)^3(0.14)^2$. Is Max correct? Explain your answer.

16 One of the terms from the expansion of $(1+3x)^7$ is chosen at random. Calculate the probability that the coefficient of this term is greater than 1000.

17 In a particular race, Carl Lewis ran the 100 m in a time of 9.99 seconds. The mean time for all athletes in the race was 10.20 seconds and the standard deviation was 0.113 seconds.
In another race Carl ran the 200 m in a time of 17.30 seconds. The mean time for this race was 18.50 seconds and the standard deviation was 0.706 seconds.

a Assuming the times in each race were normally distributed, calculate Carl's z-scores for each event.

b Based on the results of **a**, in which event did he perform better?

18 Let $f(x) = \sin(x^3)$ for $0 \leqslant x \leqslant \frac{\pi}{2}$.

a Find the x-intercepts of the graph of f.

b Sketch the graph of $y = f(x)$.

c Find the equation of the tangent to $y = f(x)$ at the point where $x = \frac{\pi}{4}$.

d There exists a point $\mathrm{P}(x, y)$ on the interval $0 \leqslant x \leqslant \frac{\pi}{2}$ such that $f(x) > 0$, $f'(x) > 0$, and $f''(x) = 0$. Find the coordinates of P.

19 A comprehensive study of a new drug for treating epilepsy was conducted during 2006-2007. The results of the treatment are shown for two age groups in the table opposite.

Treatment	Under 35	Over 35
successful	951	257
unsuccessful	174	415

a A patient from the study is selected at random. Calculate the probability that:

i the patient was successfully treated

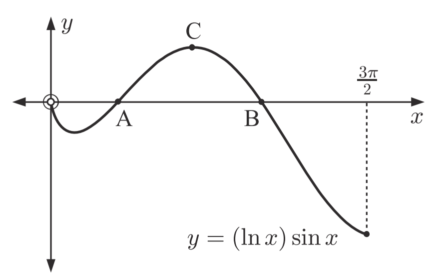
ii the patient was over 35 given that his or her treatment was unsuccessful.

b Ten patients from the study are selected at random. To calculate the probability that exactly 8 of them were successfully treated, Harry used a binomial probability distribution. Was Harry's method valid? Explain your answer.

20 Consider $f(x) = 3e^{1-4x}$. Find:

a $f'(x)$ **b** $\int f(x)\,dx$ **c** $\int_0^2 f(x)\,dx$ to 3 significant figures.

21 The graph shows $y = (\ln x)\sin x$ for $0 < x \leqslant \frac{3\pi}{2}$.
The graph crosses the x-axis at A and B, and has a local maximum at C.



a Write down exactly the x-coordinates of A and B.

b Find the coordinates of C, correct to 3 decimal places.

c Find the coordinates of the point at which the gradient of the tangent takes the largest positive value.
Give your answer correct to 3 decimal places.

d What name is given to the point in **c**?

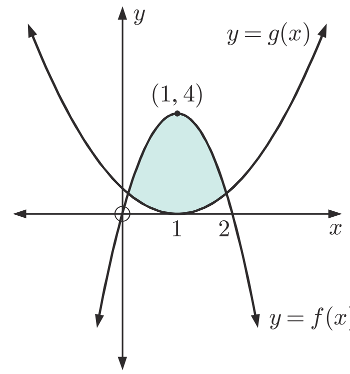
22 Mr O'Farrell deposits \$2000 into an account that pays interest at a rate of 4.5% p.a. compounded monthly. Calculate how long it will take for Mr O'Farrell's investment value to:

a double **b** quadruple.

23 **a** Find the remaining *three* terms of the expansion $(x^2+2)^5 = x^{10} + 10x^8 + 40x^6 + ...$

b Hence, find $\int (x^2+2)^5\,dx$.

24 The diagram shows the graphs of the quadratic functions $y = f(x)$ and $g(x) = (x-1)^2$.



a The graph of f has vertex $(1,\ 4)$ and x-intercepts 0 and 2.
Determine the equation of f.

b Determine the x-coordinates of the points where $y = f(x)$ and $y = g(x)$ intersect.

c Let A be the area of the region enclosed by f and g.

  i Write down an expression for A.

  ii Calculate A.

25 Events A and B are independent with $\text{P}(B) = 3\,\text{P}(A)$ and $\text{P}(A \cup B) = 0.68$.

a Show that $[\text{P}(B)]^2 - 4\,\text{P}(B) + 2.04 = 0$.

b Hence, calculate $\text{P}(B)$ and $\text{P}(A)$.

26 The weights in kilograms of twelve students are:

63 76 99 65 63 51 52 95 63 71 65 83

a Calculate the mean weight of the students.

b When one student leaves the class, the mean weight of the remaining 11 students is 70 kg. Find the weight of the student who left.

c **i** Calculate the standard deviation of the 11 remaining students.

  ii Hence, find the number of standard deviations that the heaviest student is from the mean.

27 In a mathematics quiz there are 30 multiple choice questions. There are 5 choices for each question, only one of which is correct. Assuming that you randomly guess an answer for every question, find the probability of obtaining:

a exactly 10 correct answers

b no more than 10 correct answers.

28 It is observed that 3% of all batteries produced by a company are defective.

a For a random sample of 20 batteries, calculate the probability that:

  i none are defective

  ii at least one is defective.

b Let X be the number of defectives in a random sample of n batteries.

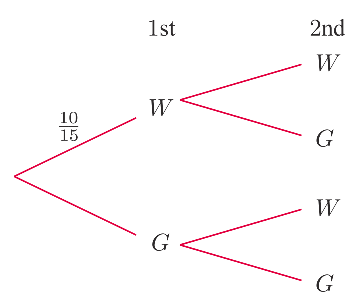
  i Write down an expression for $\text{P}(X = 0)$.

  ii Calculate the smallest value of n such that $\text{P}(X \geqslant 1) \geqslant 0.3$.

29 **a** Bag A contains 10 white and 5 green marbles. When a marble is randomly selected from the bag, W is the event that a white marble is selected, and G is the event that a green marble is selected. Two marbles are selected without replacement from Bag A.

  i Copy and complete the tree diagram.

  ii Calculate the probability that marbles of the same colour are selected.



b Bag B contains 5 white and n green marbles.
Two marbles are selected from Bag B without replacement. If the probability that both marbles are white is $\frac{2}{11}$, find n.

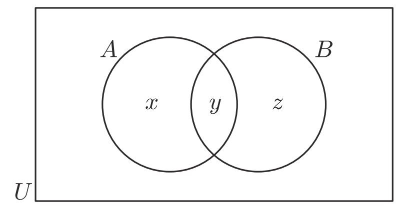
30 The mass of a sea lion on a particular island is normally distributed with mean μ and standard deviation σ. 10% of the sea lions have mass greater than 900 kg, and 15% of them have mass less than 500 kg. Find μ and σ.

31 A group of horses contract a virus at a race meeting. They are then transported back to their stable. If the virus is not detected it will spread according to the formula $H = 4 \times e^{0.3442d}$, where H is the number of horses infected after d days.

- **a** How many horses were initially infected?
- **b** The stable is home to 200 horses (including the ones initially infected). How long will it take for every horse to be infected with the virus?

32 A and B are two events such that $\text{P}(A \mid B) = 0.5$, $\text{P}(A \cup B) = 0.9$, and $\text{P}(A') = 0.2$.
For the Venn diagram given, find the probabilities:

a z **b** y **c** x.



33 Let $f(x) = \dfrac{1}{x}$, $x \neq 0$.

- **a** The graph $y = f(x)$ and the line $y = x + 2$ intersect at $m = \pm\sqrt{n}$ where $m, n \in \mathbb{Z}$. Find m and n.
- **b** The graph $y = f(x)$ is transformed to the graph $y = g(x)$ by a translation of $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$ followed by a reflection in the x-axis.
  - **i** Find an expression for $g(x)$.
  - **ii** Write down the equations of the asymptotes of $y = g(x)$.
  - **iii** Find the y-intercept of $y = g(x)$.
  - **iv** Sketch $y = g(x)$ showing the features you have found.

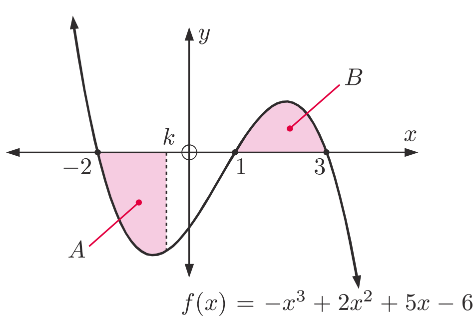
34 The position vector of a moving object is $\mathbf{r}_1 = \begin{pmatrix} -1 \\ 3 \end{pmatrix} + t\begin{pmatrix} 4 \\ 2 \end{pmatrix}$ where t is the time in seconds, $t \geqslant 0$.

A second object has a position vector $\mathbf{r}_2 = \begin{pmatrix} 0 \\ 4 \end{pmatrix} + t\begin{pmatrix} 9 \\ 4 \end{pmatrix}$.

All distances are in metres.

- **a** For the first object, determine:
  - **i** the initial position
  - **ii** the position after 10 seconds.
- **b** Hence, find the distance the first object travels in the first 10 seconds.
- **c** Show that the second object passes through the point $(9, 8)$.
- **d** Determine whether the objects collide at the point $(9, 8)$.

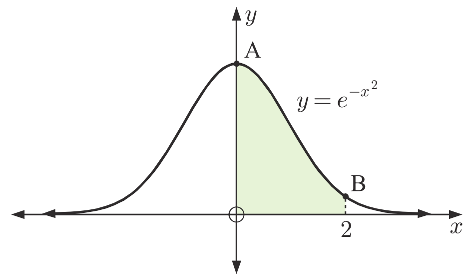
35 Consider the function $f(x) = -x^3 + 2x^2 + 5x - 6$ shown.
$x = k$ is a vertical line, where $k < 0$.
Given that the area of A equals the area of B, find the value of k.



36 The graph of $y = e^{-x^2}$ is illustrated.

a State the exact coordinates of points A and B.

b Find the area of the shaded region.



37 When a biased coin is tossed twice, the probability of getting two heads is 0.64.

a What is the probability of tossing a head with a single toss?

b If the coin is tossed 10 times, determine the probability of obtaining:

i exactly 6 heads **ii** at least 6 heads.

38 The height of a maize plant two months after planting is normally distributed with mean μ cm and standard deviation 6.8 cm. 75% of a crop of maize plants are less than 45 cm high. Suppose X describes the height of a maize plant.

a Find the mean height μ cm. **b** Find $\text{P}(X < 25)$.

c Find a such that $\text{P}(X < 25) = \text{P}(X > a)$.

39 **a** On the same set of axes, graph $y = 3e^{-x}$ and $y = e^{2x} + 1$.

b Hence solve $e^{2x} + 1 = 3e^{-x}$ for x. Give your answer correct to 4 decimal places.

40 Let $f(x) = x\sin(2x)$, $0 < x < 3$.

a Sketch the graph of $y = f(x)$.

b Find the range of $f(x)$.

c Find the x-intercept b of the graph of $y = f(x)$ on the given domain.

d The region enclosed by $y = f(x)$ and the x-axis from $x = 0$ to $x = b$, has area A units^2. Find A correct to 3 decimal places.

41 Consider $f(x) = e^{-3x}\sin x$, $-\frac{1}{2} \leqslant x \leqslant 3$.

a Show that $f'(x) = e^{-3x}(\cos x - 3\sin x)$.

b Find the equation of the tangent to the curve $y = f(x)$ at the point $\text{P}(\frac{\pi}{2}, e^{-\frac{3\pi}{2}})$.

c Find the area between the curve and the x-axis from $x = 0$ to $x = 1$.

42 Consider $f(x) = \dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}$, $0 \leqslant x \leqslant \frac{\pi}{2}$.

a Show that $f(x) = \dfrac{2}{\sin 2x}$.

b Solve the equation $\sin 2x = 0$ and hence state the equations of any asymptote of $y = f(x)$ on $0 \leqslant x \leqslant \frac{\pi}{2}$.

c Without using calculus, find the least value of $f(x)$ and the corresponding value of x.

d If $\sin\alpha = \frac{1}{3}$, find $f(2a)$ correct to 4 significant figures.

43 On an ostrich farm the weights of the birds are found to be normally distributed. The weights of the females have mean 78.6 kg and standard deviation 5.03 kg. The weights of the males have mean 91.3 kg and standard deviation 6.29 kg.

a Find the probability that a randomly selected:

i male will weigh less than 80 kg **ii** female will weigh less than 80 kg

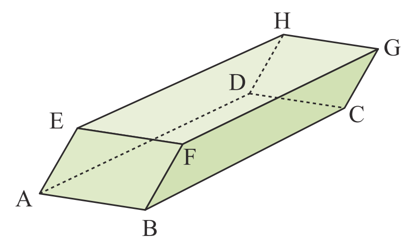
iii female will weigh between 70 and 80 kg.

b 20% of females weigh less than k kg. Find k.

c The middle 90% of the males weigh between a kg and b kg. Find the values of a and b.

d In one field there are 82% females and 18% males. One of these ostriches is selected at random. Calculate the probability that the ostrich weighs less than 80 kg.

44 The solid figure shown is a parallelepiped. All six of its faces are parallelograms.
A is $(3, 2, -1)$, B is $(1, -1, 4)$, and C is $(2, 0, 7)$.



a Find $\overrightarrow{\mathrm{BA}}$ and $|\overrightarrow{\mathrm{BA}}|$.

b Find the coordinates of D.

c If $\overrightarrow{\mathrm{BF}} = \begin{pmatrix} 6 \\ -3 \\ -6 \end{pmatrix}$, find the coordinates of F.

d Calculate $\overrightarrow{\mathrm{BA}} \bullet \overrightarrow{\mathrm{BF}}$ and hence find $\cos \mathrm{A\widehat{B}F}$.

e Hence, find the exact area of parallelogram ABFE.

45 A discrete random variable Y has the probability distribution shown:

y	1	2	3	4	5
$\mathrm{P}(Y = y)$	$\frac{1}{10}$	$2t$	$\frac{3}{20}$	$2t^2$	$\frac{t}{2}$

a Find t.

b Find the expected value of the random variable Y.

c Explain the significance of your result in **b**.

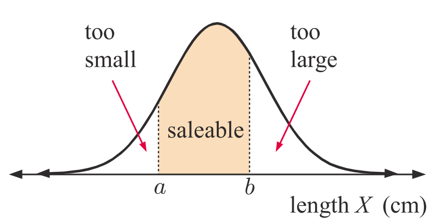
46 Consider $f(x) = 5x + e^{1-x^2} - 2$ for $-1 \leqslant x \leqslant 2$.

a Find the y-intercept of f. **b** Sketch the graph of $y = f(x)$.

c Find any x-intercepts of f.

d Find the gradient of the tangent to $y = f(x)$ at the point where $x = 1$.

47 The length of a zucchini is normally distributed with mean 24.3 cm and standard deviation 6.83 cm. A supermarket buying zucchinis in bulk finds that 15% of them are too small and 20% of them are too large for sale. The remainder, with lengths between a cm and b cm, are able to be sold.



a Find the values of a and b.

b A zucchini is chosen at random. Find the probability that:

i it is of saleable length **ii** its length lies between 20 and 26 cm

iii its length is less than 24.3 cm.

48 **a** Sketch the graph of $y = \dfrac{6-2x}{x+3}$.

b Discuss the behaviour of the graph near its asymptotes, and hence deduce their equations.

c State the values of $\lim\limits_{x\to-\infty} \dfrac{6-2x}{x+3}$ and $\lim\limits_{x\to\infty} \dfrac{6-2x}{x+3}$.

49 Line L_1 has vector equation $\mathbf{r}_1 = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + s\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$.

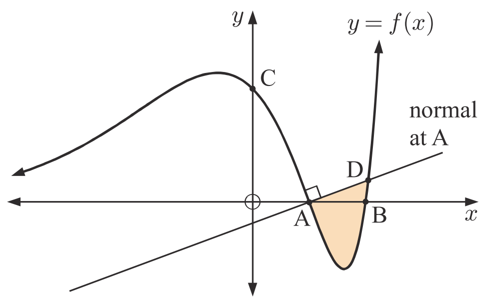
Line L_2 has vector equation $\mathbf{r}_2 = \begin{pmatrix} 6 \\ -3 \\ 4 \end{pmatrix} + t\begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$.

a Point A lies on L_1 and has z-coordinate -2. Find the coordinates of A.

b Do lines L_1 and L_2 intersect? If they do, where do they meet?

c Find, correct to one decimal place, the angle between L_1 and L_2.

50 The graph of $f(x) = e^x(x^2-3x+2)$ is shown.



a Find the coordinates of points A, B, and C.

b Write down the equations of any asymptotes of $y = f(x)$.

c Show that $f'(x) = e^x(x^2-x-1)$.

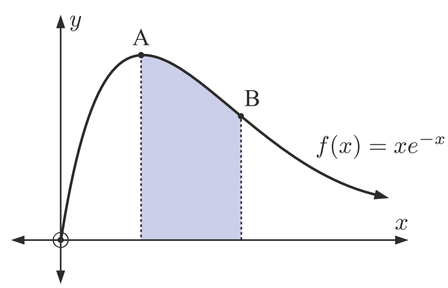
d Find the x-coordinates of the local maximum and local minimum.

e Show that the normal to the curve at A has equation $x - ey = 1$.

f Find the x-coordinate of D correct to 3 significant figures.

g Hence estimate the area of the shaded region.

51 The graph of $f(x) = xe^{-x}$, $x \geqslant 0$ is shown.



a Find the y-intercept of f.

b Find $f'(x)$ and hence find the coordinates of the local maximum A.

c Find exactly the x-coordinate of the point of inflection B.

d Find the area of the shaded region.

52 When a pair of normal dice are rolled, E is the event of rolling *at least one* 6.

a Display the possible results on a 2-dimensional grid.

b Determine $P(E)$.

c The two dice are rolled 10 times. Determine the probability that event E occurs:

i exactly 2 times **ii** at most 3 times.

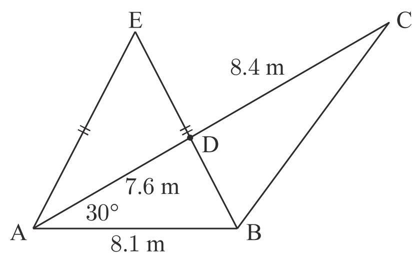
53 Points A and B have coordinates $(2, -1, 3)$ and $(-2, 4, -1)$ respectively.

a Show that the line L_1 which passes through A and B has vector equation
$$\mathbf{r}_1 = (2 - 4t)\mathbf{i} + (5t - 1)\mathbf{j} + (3 - 4t)\mathbf{k}.$$

b $C(4, a, b)$ lies on the line through A and B. Find a and b.

c L_2 has equation $\mathbf{r}_2 = (3s - 5)\mathbf{i} + (s - 16)\mathbf{j} + (16 - s)\mathbf{k}$.
Find the coordinates of the point where L_1 and L_2 intersect.

54



Consider the figure shown.

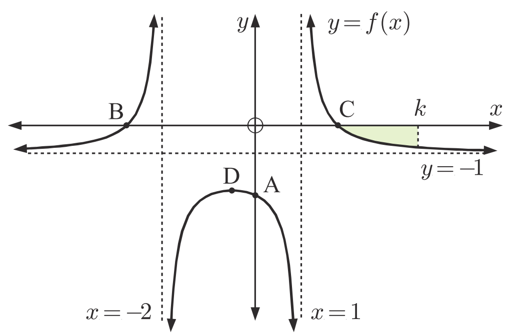
a Find the lengths of [DB] and [BC].

b Calculate the measures of $A\widehat{B}E$ and $D\widehat{B}C$.

c Find the area of $\triangle$BCD.

d Calculate the length of [AE].

55 The graph of $f(x) = a + \dfrac{3}{(x-1)(x+b)}$ where $a, b \in \mathbb{Z}$ is shown.



a State the values of a and b.

b Find the y-intercept.

c Find exactly the x-intercepts.

d Show that $f'(x) = \dfrac{-3(2x+1)}{(x^2+x-2)^2}$ and hence find the coordinates of the local maximum D.

e **i** Write down an expression for the shaded area as an integral.

ii Find the shaded area for $k = 3$.

56 G is the event of a customer buying a goldfish from a pet shop and T is the event of a customer buying a tortoise from a pet shop.

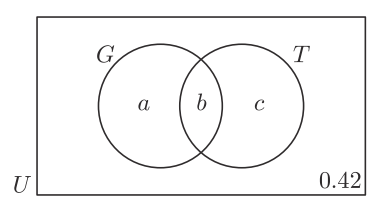
a At one particular pet shop, $P(G) = 0.3$, $P(T) = 0.4$, and $P(\text{neither } G \text{ nor } T) = 0.42$.

i Find $P(G \cap T)$. Explain what this result means.

ii Show this information on a Venn diagram.

iii Are G and T independent events? Give reasons for your answer.

b The probabilities at a second pet shop are represented in the Venn diagram.



$P(T)$ is twice $P(G)$, $P(\text{neither } G \text{ nor } T) = 0.42$, and G and T are independent.

i Show that $2a + b = c$.

ii Show that $a = \sqrt{\dfrac{b}{2}} - b$.

iii Hence, find b and then a.

iv Determine $P(G)$.

57 The weight of DDT in a pile of contaminated soil is given by $W_t = 1800 \times (0.95)^t$ grams, where t is the number of years since the soil was contaminated.

a Find the initial quantity of DDT which contaminated the soil.

b Find the quantity of DDT remaining after:

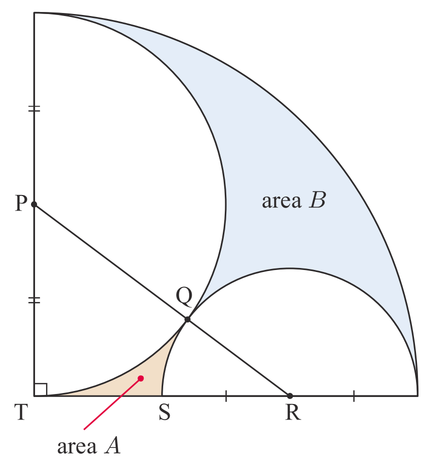
i $7\frac{1}{2}$ years **ii** 40 years.

c Sketch the graph of W_t against t.

d The *half-life* of DDT is the time it takes for the quantity of DDT to decrease to half of the initial amount.
Use your graph in **c** to estimate the half-life of DDT.

e Find the time required until 100 g of DDT remains in the soil.

58 Two semi-circles touch each other within a quarter circle as shown. This means that P, Q, and R are collinear. The radius of the quarter circle is 12 cm.



a Use the theorem of Pythagoras to show that the radius of the smaller semi-circle is 4 cm.

b Calculate, in radians, the measure of:

i $T\widehat{P}R$ **ii** $P\widehat{R}T$

c Hence calculate the area of:

i A **ii** B

59 Solve the following equations:

a $\log_2(x^2 - 2x + 1) = 1 + \log_2(x - 1)$ **b** $3^{2x+1} = 5(3^x) + 2$

60 A zoo's population of pygmy shrews is weighed during the annual veterinary health check.

Length (mm)	95	83	91	82	75	62	79	63	81	69	94	88	72	77
Weight (g)	5.4	4.5	5.0	4.1	3.7	2.6	4.5	3.1	4.7	3.7	5.1	4.8	3.6	4.2

a Draw a scatter diagram for this data.

b Calculate Pearson's product-moment correlation coefficient r for the data.

c Hence describe the correlation between these two variables.

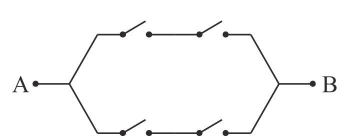
d Find the equation of the least squares regression line.

e Hence predict the weight of a pygmy shrew with length:

i 110 mm **ii** 70 mm

f Which of your predictions in **e** is more likely to be reliable? Explain your answer.

61 The diagram shows a simple electrical network.



Each symbol —•⁄•— represents a switch.

All four switches operate independently, and the probability of each one of them being closed is p.

a In terms of p, find the probability that the current flows from A to B.

b Find the least value of p for which the probability of current flow is at least 0.5.

62 In triangle ABC, the angle at A is double the angle at B. AC = 5 cm and BC = 6 cm.

- **a** Find the cosine of the angle at B.
- **b** Find the length of [AB] using the sine rule.
- **c** If the cosine rule was used to find the length of [AB], how many solutions would it give?

63 ABC is an equilateral triangle with sides 10 cm long. P is a point within the triangle which is 5 cm from A and 6 cm from B. How far is P from C?

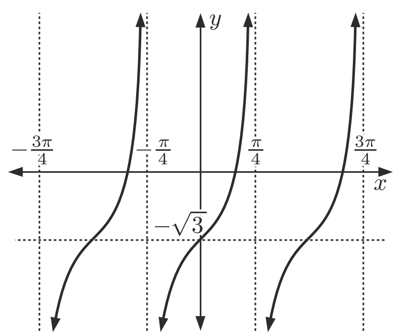
64 A company manufactures computer chips. It is known that 3% of them are faulty. In a batch of 500 chips, find the probability that between 1 and 2 percent (inclusive) of the chips are faulty.

65 A random variable X is normally distributed with standard deviation 2.83. Find the probability that a randomly selected score from X will differ from the mean by less than 4.

66 The Ferris wheel at the Royal Show turns one full circle every minute. The lowest point is 1 metre from the ground, and the highest point is 25 metres above the ground.

- **a** When riding on the Ferris wheel, your height above ground level after t seconds is given by the model $h(t) = a + b\sin(c(t-d))$. Find the values of a, b, c, and d given that you start your ride at the lowest point.
- **b** If the motor driving the Ferris wheel breaks down after 91 seconds, how high up would you be while waiting to be rescued?

67 Consider the graph of $y = \tan ax + b$ shown.



- **a** Find the values of a and b.
- **b** Hence, find the x-intercepts of the function, for $-\frac{3\pi}{4} \leqslant x \leqslant \frac{3\pi}{4}$.

68 The function f is defined by $f : x \mapsto e^{\sin^2 x}$, $0 \leqslant x \leqslant \pi$.

- **a** Use calculus to find the exact value(s) of x at which $f(x)$ has a maximum turning point.
- **b** Find $f''(x)$.
- **c** Find any points of inflection in the given domain.

69 The sum of an infinite geometric series is 49. The second term of the series is 10. Find the possible values for the sum of the first three terms of the series.

70 Let $f(x) = xe^{1-2x^2}$.

- **a** Find $f'(x)$ and $f''(x)$.
- **b** Find the exact coordinates of the stationary points of the function and determine their nature.
- **c** Find exactly the x-coordinates of the points of inflection of the function.
- **d** Discuss the behaviour of the function as $x \to \pm\infty$.
- **e** Sketch the function, showing the information you have found.

71 Consider the grouped data in the table. Estimate:

a the mean value of X

b the standard deviation of the X-distribution.

Score	*Frequency*
$9 \leqslant X < 11$	2
$11 \leqslant X < 13$	7
$13 \leqslant X < 15$	6
$15 \leqslant X < 17$	21
$17 \leqslant X < 19$	17
$19 \leqslant X < 21$	5

72 Consider the vectors $\mathbf{a} = \begin{pmatrix} \cos\theta \\ \sin\theta \\ \cos\theta \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} \sin\theta \\ -\sin\theta \\ \cos\theta \end{pmatrix}$.

a Show that $|\mathbf{a}| = \sqrt{1+\cos^2\theta}$.

b Explain why $1 \leqslant |\mathbf{a}| \leqslant \sqrt{2}$ for all θ.

c Find all values of θ in the interval $0 \leqslant \theta \leqslant 2\pi$ such that **a** and **b** are perpendicular.

73 Solve for x on the domain $0 \leqslant x \leqslant 2\pi$:

a $3 + 2\sin x = 0$

b $3\cos\left(\frac{x}{2}\right) + 1 = 0$

74 A random variable X has the discrete probability density function $\text{P}(X = x) = \dfrac{x^2 + kx}{50}$ for $x = 0, 1, 2, 3, 4$.

a Find k.

b Find μ, the mean of the distribution of X.

c Find $\text{P}(X \geqslant 2)$.

75 Consider $f(x) = \sin x \cos(2x)$ for $0 \leqslant x \leqslant \pi$.

a Find $f'(x)$ in terms of $\cos x$ only.

b Show that $f'(x) = 0$ when $\cos x = 0$ or $\pm\sqrt{\frac{5}{6}}$.

c Hence, find the position and nature of the turning points of $y = f(x)$.

d Graph $y = f(x)$, showing the features you have found.

76 An experiment was conducted to measure the shelf life of bottles of milk stored at different temperatures:

Temperature T (°C)	1	4	10	12	15	21	27
Shelf life D (days)	19	11.5	4	3	1.5	0.5	0.25

a Draw scatter diagrams of:

i D against T **ii** $\ln D$ against T **iii** $\ln D$ against $\ln T$

b Determine which of the diagrams illustrates a linear relationship, and find the equation of the least squares regression line in this case.

c Hence determine a formula for D in terms of T.

d Estimate the shelf life of a bottle of milk stored at 7°C.