Chapter 25

Miscellaneous questions

Contents:

A NON-CALCULATOR QUESTIONS

EXERCISE 25A

1 A geometric sequence has $S_1 = 2$ and $S_2 = 8$. Find:

a the common ratio r **b** the twentieth term u_{20}.

2 Show that the sum of the first forty terms of the series $\ln 2 + \ln 4 + \ln 8 + \ln 16 + \ldots$ is $820 \ln 2$.

3 Suppose $f(x) = be^x$ and $g(x) = \ln(bx)$. Find:

a $(f \circ g)(x)$ **b** $(g \circ f)(x)$

c an expression for x^*, in terms of b, such that $(f \circ g)(x^*) = (g \circ f)(x^*)$.

4 Consider $f(x) = -2(x-b)^2 + 2$.

a State the coordinates of the vertex.

b Find the axes intercepts.

c The graph of function g is obtained by translating the graph of f vertically through b units. For what values of b will the graph of g:

i have exactly one x-intercept **ii** have no x-intercepts

iii pass through the origin?

5 **a** Expand $(x-2)^3$.

b Hence, find the coefficient of x^3 in $(3x^2 - 7)(x-2)^3$.

6 Consider $f(x) = \sqrt{1-2x}$. Find:

a $f(0)$ **b** $f(-4)$ **c** the domain of f **d** the range of f.

7 Let $a = \sin 20°$ and $b = \tan 50°$. In terms of a and b, write expressions for:

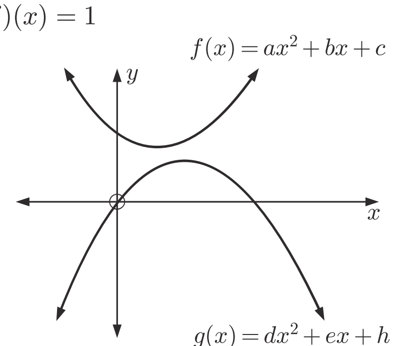
a $\sin 160°$ **b** $\tan(-50°)$ **c** $\cos 70°$ **d** $\tan 20°$

8 Suppose $f(x) = \cos x$ and $g(x) = 2x$.
Solve the following equations on the domain $0 \leqslant x \leqslant 2\pi$:

a $(f \circ g)(x) = 1$ **b** $(g \circ f)(x) = 1$

9 Consider the graphs illustrated.
Copy and complete the following table by indicating whether each constant is positive, negative, or zero:

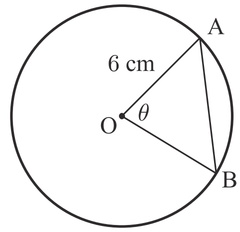
Constant	a	b	c	d	e	h	Δ of $f(x)$	Δ of $g(x)$
Sign								



10 In the given figure, the perimeter of sector AB is $(12 + 2\pi)$ cm.

a Find the value of θ.

b Hence state the length of chord AB.



11 Let $f(x) = x^2 + 6$.

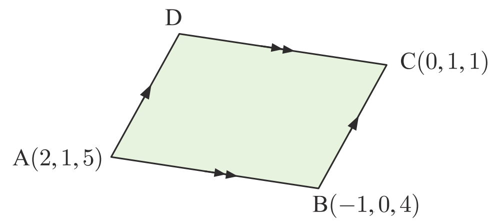
a Can you solve $f(x) = 3$? **b** What does this tell us about the range of $f(x)$?

12 Consider $\mathbf{u} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $\mathbf{v} = 3\mathbf{i} + p\mathbf{j} - \mathbf{k}$.

a Find p if $\mathbf{u}$ is perpendicular to $\mathbf{v}$. **b** Evaluate $|\mathbf{u}|\,|\mathbf{v}|$.

c Show that no value of p exists such that $\mathbf{v} - \mathbf{u}$ and $\mathbf{u}$ are parallel.

13 Consider the parallelogram ABCD illustrated.



a Find $\overrightarrow{BA}$ and $\overrightarrow{BC}$.

b Hence find $|\overrightarrow{BA}|$ and $|\overrightarrow{BC}|$.

c What can be deduced about parallelogram ABCD?

d Calculate:

i $\cos(C\widehat{B}A)$ **ii** $\sin(C\widehat{B}A)$ **iii** the area of ABCD.

14 $a, b, c, d, e, f, g, h, i, j, k, l,$ and m are 13 data values which have been arranged in *ascending* order.

a Which variable represents the median?

b Write down an algebraic expression for:

i the range **ii** the interquartile range.

15 For the data set $\{a, b, c\}$, the mean is 17.5 and the standard deviation is 3.2.

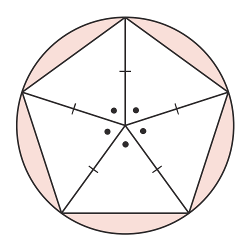
Copy and complete the following table by finding the mean and standard deviation of each new data set:

	New data set	*Mean*	*Standard deviation*
a	$\{2a, 2b, 2c\}$		
b	$\{a+2, b+2, c+2\}$		
c	$\{3a+5, 3b+5, 3c+5\}$		

16 A dart is thrown at the dartboard shown. It is equally likely to land anywhere on the board.

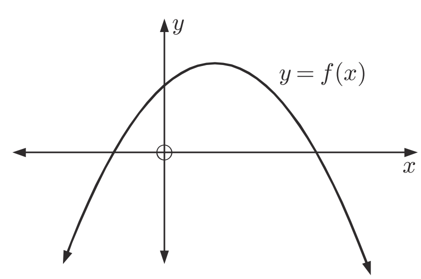
Given that the dart lands on the board, show that the probability of it landing on the shaded region is exactly

$$1 - \frac{5}{2\pi}\sin\left(\frac{2\pi}{5}\right).$$



17 For the function $y = f(x)$ with graph shown, sketch the graphs of:

a $y = f'(x)$ **b** $y = f''(x)$



18 Consider $g(x) = 3 - 2\cos(2x)$.

a Find $g'(x)$.

b Sketch $y = g'(x)$ for $-\pi \leqslant x \leqslant \pi$.

c Write down the number of solutions to $g'(x) = 0$ for $-\pi \leqslant x \leqslant \pi$.

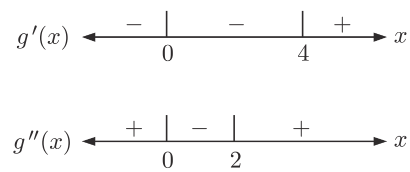
d Mark a point M on the sketch in **b** where $g'(x) = 0$ *and* $g''(x) > 0$.

19 A and B are mutually exclusive events where $\text{P}(A) = x$ and $\text{P}(B') = 0.43$.

a Write $\text{P}(A \cup B)$ in terms of x.

b Find x given that $\text{P}(A \cup B) = 0.73$.

20 For the function $g(x)$, the sign diagrams for $g'(x)$ and $g''(x)$ are shown alongside.
The points A(0, 2), B(2, 0), and C(4, −2) all lie on $y = g(x)$.
Sketch $y = g(x)$, labelling the stationary points.



21 Consider $f(x) = xe^{1-2x}$.

a Show that $f'(x) = e^{1-2x}(1 - 2x)$.

b Find the point on the graph of $y = f(x)$ where the tangent is horizontal.

c Find values of x for which:

i $f(x) > 0$

ii $f'(x) > 0$

22 A particle moves in a straight line so that its position s at time t seconds is given by $s(t) = 3 - 4e^{2t} + kt$ metres, where k is a constant.

a Find the velocity function $v(t)$ in terms of k.

b Determine the value of k, given the particle is stationary when $t = \ln 3$ seconds.

23 Solve for x:

a $\log_3 27 = x$

b $e^{5-2x} = 8$

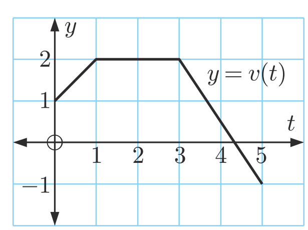
c $\ln(x^2 - 3) - \ln(2x) = 0$

24 The graph shows the velocity v m s^{-1} of an object at time t seconds, $t \geqslant 0$. Find and interpret:

a $v(0)$

b $v'(2)$

c $\displaystyle\int_1^3 v(t)\,dt$.



25 Suppose $\displaystyle\int_{-1}^{2} f(x)\,dx = 10$. Find the value of:

a $\displaystyle\int_{-1}^{2} (f(x) - 6)\,dx$

b k if $\displaystyle\int_{2}^{-1} k\,f(x)\,dx = -5$.

26 **a** Show that $(\sin\theta - \cos\theta)^2 = 1 - \sin 2\theta$.

b Hence calculate $\displaystyle\int_0^{\frac{\pi}{4}} (\sin\theta - \cos\theta)^2\,d\theta$.

27 The following table shows the probability distribution for a discrete random variable X.

x	-2	-1	0	1	2
$\text{P}(X = x)$	0.1	a	0.25	b	0.15

Find a and b given that $\text{E}(X) = 0$.

28 Consider the infinite geometric sequence: $e, \frac{1}{e}, \frac{1}{e^3}, \frac{1}{e^5}, ...$ Find, in terms of e:

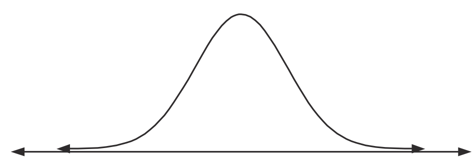
a the common ratio
b the 101st term
c the sum of the corresponding infinite series.

29 **a** Use the formula $\binom{n}{r} = \dfrac{n!}{r!\,(n-r)!}$ to evaluate $\binom{6}{2}$.

b Hence state the value of $\binom{6}{4}$.

c Given that $\binom{6}{3} = 20$, write down the expansion of $(x-2)^6$. Simplify your answer.

30 The random variable X is normally distributed with mean μ and standard deviation σ.
Let k be such that $\text{P}(X < k) = 0.7$.



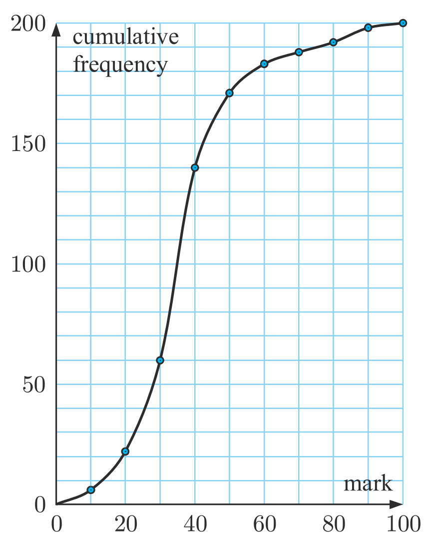
a Copy the normal distribution curve and mark on it μ and k.

b On the graph shade the region which illustrates $\text{P}(X < k) = 0.7$.

c Find:
i $\text{P}(X > k)$
ii $\text{P}(\mu < X < k)$
iii $\text{P}(\mu - \sigma < X < k)$

d If $\text{P}(X \geqslant t) = 0.2$, find $\text{P}(k \leqslant X \leqslant t)$.

31



The examination marks for 200 students are displayed on the cumulative frequency graph shown. The pass mark for the examination was 30.

a What percentage of the students passed the examination?

b A boxplot for the examination data is:

From the graph, estimate:
i m
ii n
iii p
iv q

32 The line L has equation $y = \left(\tan \frac{\pi}{3}\right) x$.

a Find p given that the point $\text{A}(10, p)$ lies on L.

b Find the equation of the line which passes through A and is perpendicular to L.
Write your answer in the form $ax + by = c$.

33 The acceleration of an object moving in a straight line is given by
$a(t) = 1 - 3\cos\left(2t + \frac{\pi}{2}\right)$ cm s^{-2}, where t is the time in seconds.
The object's initial velocity is 5 cm s^{-1}.

a Find an expression for the object's velocity v in terms of t.
b Find the velocity of the object at $t = \frac{\pi}{4}$ seconds.

34 Consider $f(x) = e^{3x-4} + 1$.

a Show that $f^{-1}(x) = \dfrac{\ln(x-1)+4}{3}$.
b Calculate $f^{-1}(8) - f^{-1}(3)$. Give your answer in the form $a \ln b$ where $a, b \in \mathbb{Q}^+$.

35 Given that $\sin A = \frac{2}{5}$ and $\frac{\pi}{2} \leqslant A \leqslant \pi$, find:

a $\cos A$ **b** $\tan A$ **c** $\sin 2A$

36 Consider the infinite geometric sequence $160, 80\sqrt{2}, 80, 40\sqrt{2}, ...$

a Write the 12th term of the sequence in the form $k\sqrt{2}$ where $k \in \mathbb{Q}$.
b Find exactly the following sums for the corresponding geometric series. Give your answer in the form $a + b\sqrt{2}$ where $a, b \in \mathbb{Z}$:
i the partial sum S_{10} **ii** the sum S of the infinite series.

37 A particle is initially located at P(3, 1, -2). It moves with fixed velocity in a straight line, and after 2 seconds the particle is at Q(1, 3, 4). Find:

a $\overrightarrow{PQ}$ **b** the particle's speed **c** the equation of the straight line.

38 A journalist is investigating the consistency of online reviews for electrical items. She compares the scores given to two different camera models by 6 different reviewers, recording the results in the table shown.

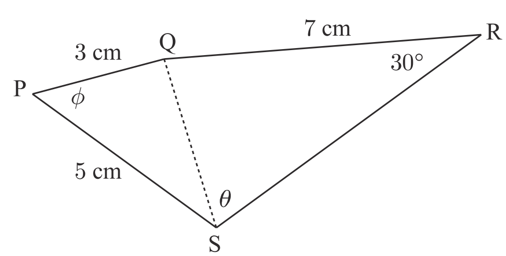
Camera A	8.5	8	9	7	8.5	7.5
Camera B	7	6	7.5	9	7.5	6

a Draw a scatter diagram of the data. **b** Are there any outliers?
c What can the journalist report about the consistency of online reviews for these cameras?

39 Suppose $g(x) = e^{\frac{x}{4}}$ where $0 \leqslant x \leqslant 4$.

a Sketch $y = g(x)$ on the given domain. **b** Find the range of $g(x)$.
c On the same set of axes used in **a**, sketch $y = g^{-1}(x)$.
d State the domain and range of $g^{-1}(x)$. **e** Find $g^{-1}(x)$ algebraically.

40 [QS] is a diagonal of quadrilateral PQRS where PQ = 3 cm, QR = 7 cm, PS = 5 cm, and $\widehat{QRS} = 30°$.



a Show that if $\widehat{SPQ} = \phi$, then QS $= \sqrt{34 - 30\cos\phi}$ cm.
b If $\phi = 60°$ and $\widehat{QSR} = \theta$:
i show that $\sin\theta = \frac{7}{2\sqrt{19}}$
ii find the *exact* length of [RS], given that θ is acute
iii hence, find the exact perimeter and area of PQRS.

41 Suppose $f(x) = -\frac{1}{4}x^2 + 3x + 4$.

a Find $f'(x)$ in simplest form.

b **i** Find the equation of the normal to $y = f(x)$ at the point (2, 9).

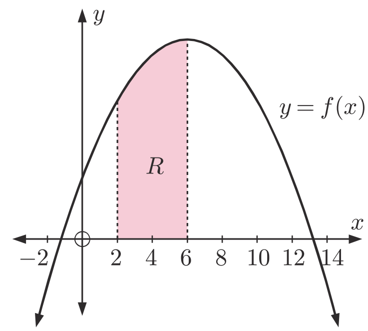
ii Find the coordinates of the point where this normal meets $y = f(x)$ again.

c The graph of $y = f(x)$ is shown alongside.

i Write down an expression for the area of the shaded region R.

ii Calculate the exact area of R.

iii Suppose the region R is revolved about the x-axis through one revolution. Find an expression for the volume of the solid formed.



42 **a** Consider the geometric sequence: 4, −12, 36, −108, ...

i Write down the common ratio.

ii Find the 14th term.

b Suppose the sequence x, $x - 2$, $2x - 7$, ... is geometric.

i Find x.

ii Does the sum of the corresponding geometric series converge? Explain your answer.

c Suppose the sequence x, $x - 2$, $2x - 7$, ... is arithmetic. Find:

i its 30th term

ii the sum of its first 50 terms.

43 Suppose A has position vector $3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, B has position vector $2\mathbf{i} - \mathbf{j} - 8\mathbf{k}$, and C has position vector $\mathbf{i} + \mathbf{j} + a\mathbf{k}$.

a Find $\overrightarrow{AB}$.

b Find the unit vector $\mathbf{u}$ in the direction of $\overrightarrow{BA}$.

c Is $\mathbf{u}$ perpendicular to $\overrightarrow{OA}$?

d Find a given that $\overrightarrow{OC}$ is perpendicular to $a\mathbf{i} - \mathbf{j} + 4\mathbf{k}$.

e Find the position vector of M, the midpoint of [AB].

f Line L_1 passes through M and is parallel to $\overrightarrow{OA}$.
Write down the vector equation $\mathbf{r}_1$ of line L_1.

g Suppose line L_2 has vector equation $\mathbf{r}_2 = (m\mathbf{i} + \mathbf{j} - \mathbf{k}) + s(2\mathbf{i} - 3\mathbf{j} + \mathbf{k})$.

i Explain why L_1 is not parallel to L_2.

ii Find m if L_1 and L_2 intersect.

iii Find the position vector of P, the point of intersection of L_1 and L_2.

44 Consider the functions $f(x) = \frac{1}{2}x - 1$ and $g(x) = \sqrt{3}x$.
Find exactly the angle between the straight line and the positive x-axis for:

a $y = f(x)$

b $y = (f \circ g)x$.

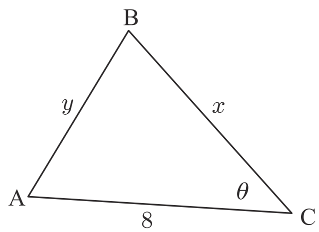
45 **a** Consider the quadratic function $y = -x^2 + 12x - 20$.

i Explain why this quadratic has a maximum value.

ii What value of x gives this maximum value?

iii What is the maximum value?

b



In $\triangle$ABC, $\text{AB} = y$, $\text{BC} = x$, and $\text{AC} = 8$. The perimeter of $\triangle$ABC is 20.

i Write y in terms of x.

ii Use the cosine rule to write y^2 in terms of x and $\cos\theta$.

iii Hence, show that $\cos\theta = \dfrac{3x-10}{2x}$.

iv If the area of the triangle is A, show that $A^2 = 16x^2 \sin^2\theta$.

v Show that $A^2 = 20(-x^2 + 12x - 20)$.

vi Hence, find the maximum area of $\triangle$ABC.

vii Comment on the shape of the triangle when it has maximum area.

46 Suppose $f(x) = 4x - 3$ and $g(x) = x + 2$.

a Find $f^{-1}(x)$ and $g^{-1}(x)$, the inverse functions of f and g.

b Find $(f \circ g^{-1})(x)$.

c Find the value of x such that $(f \circ g^{-1})(x) = f^{-1}(x)$.

d Suppose $H(x) = \dfrac{f(x)}{g(x)}$.

i Sketch the graph of $y = H(x)$. Include its asymptotes and their equations.

ii Find constants A and B such that $\dfrac{4x-3}{x+2} = A + \dfrac{B}{x+2}$.

iii Calculate the exact value of $\displaystyle\int_{-1}^{2} H(x)\,dx$.

iv On your sketch in **i**, shade the region whose area is given by $\displaystyle\int_{1}^{3} H(x)\,dx$.

47 Hannah, Heidi, and Holly have different sets of cards, but each set contains cards with the numbers 0, 1, 2, 3, or 4, one per card.

a Hannah wrongly states that the probability distribution of her set of cards is:

x	0	1	2	3	4
$\text{P}(X = x)$	0.1	0.3	0.3	0.2	0.2

Why is Hannah wrong?

b Heidi correctly states that her probability distribution is:

x	0	1	2	3	4
$\text{P}(X = x)$	0.2	a	0.3	b	0.2

What can be deduced about a and b?

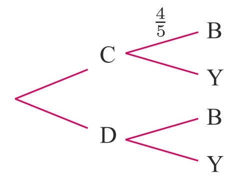
c Holly correctly states that the probability distribution for her set of cards is

$$\text{P}(X = x) = \frac{x(x+2)}{50}.$$

If one card is randomly chosen from Holly's set, find the probability that it is:

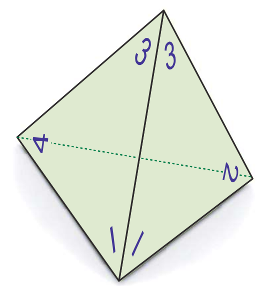
i a 2 **ii** not a 2.

48 Bag C contains 4 blue and 1 yellow ticket. Bag D contains 2 blue and 3 yellow tickets.
An ordinary 6-sided die is used to select one of the two bags.
If a 1 or 2 is rolled, bag C is chosen. Otherwise, bag D is chosen.
A ticket is drawn at random from that bag.



- **a** Copy and complete the tree diagram, showing all probabilities.
- **b** Find the probability that a yellow ticket is drawn from bag D.
- **c** Find the probability of drawing a yellow ticket from either bag.
- **d** If a blue ticket is chosen, find the probability that it came from bag D.
- **e** In a gambling game, a player wins \$6 for getting a blue ticket and \$9 for getting a yellow one. Find the player's expected return.

49 Two identical tetrahedral dice are rolled. Their four vertices are clearly labelled 1, 2, 3, and 4.
The result when one die comes to rest is the number on the uppermost vertex. This is 3 in the diagram.



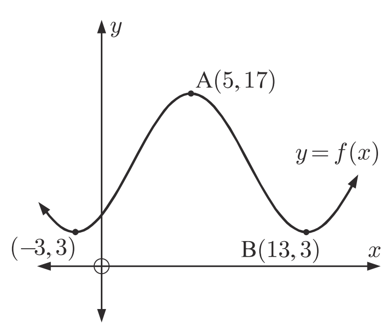
- **a** Illustrate the sample space of 16 possible results when the two dice are rolled.
- **b** Let X be the sum of the scores on the two dice.
  What are the possible values of X?
- **c** Find: **i** $P(X = 4)$ **ii** $P(X > 4)$
- **d** When Mimi uses the two dice to play a fair game, she:
  - wins €5 if the sum is 4
  - wins €1 if the sum is greater than 4
  - loses €d if the sum is less than 4.

  Find the value of d.

50 A particle moves in a straight line such that at time t seconds, $t \geqslant 0$, the acceleration is $a(t) = 3t - \sin t \ \text{cm s}^{-2}$.

- **a** Find the particle's acceleration at times $t = 0$ and $t = \frac{\pi}{2}$ seconds.
- **b** If the initial velocity of the particle is 3 cm s^{-1}, find its velocity function $v(t)$.
- **c** Find $\int_0^{\frac{\pi}{2}} v(t)\, dt$ and explain why the result is positive.
- **d** Interpret the result in **c** with regard to the particle's motion.

51 The graph of $f(x) = a\sin b(x - c) + d$ is illustrated.
A is a local maximum and B is a local minimum.



- **a** Find the values of a, b, c, and d.
- **b** The function $g(x)$ is obtained from $f(x)$ by a translation of $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ followed by a vertical stretch with scale factor 2.
  - **i** Find the coordinates of A′, the image of A under T.
  - **ii** Find $g(x)$ in the form $g(x) = p\sin q(x - r) + s$.
  - **iii** Describe fully the transformation which maps g back to f.

52 **a** Factorise $4^x - 2^x - 20$ in the form $(2^x + a)(2^x - b)$ where $a, b \in \mathbb{Z}^+$.

b Hence, find the exact solution of $2^x(2^x - 1) = 20$.

c Suppose $p = \log_5 2$.

i Write the solution to **b** in terms of p.

ii Find the solution to $8^x = 5^{1-x}$ in terms of p only.

53 Suppose $f(x) = a\cos 2x + b\sin^2 x$ where $b < 2a$, $0 \leqslant x \leqslant 2\pi$.

a Show that $f'(x) = (b - 2a)\sin 2x$.

b Find the maximum value of $f'(x)$ and when this maximum occurs.

c Find the turning points of $y = f(x)$ on $0 \leqslant x \leqslant 2\pi$.

54 Suppose $S(x) = \frac{1}{2}(e^x - e^{-x})$ and $C(x) = \frac{1}{2}(e^x + e^{-x})$.

a Show that $[C(x)]^2 - [S(x)]^2 = 1$.

b Show that $\frac{d}{dx}[S(x)] = C(x)$.

c Find $\frac{d}{dx}[C(x)]$ in terms of $S(x)$.

d If $T(x) = \frac{S(x)}{C(x)}$, find $\frac{d}{dx}[T(x)]$ in terms of $C(x)$.

55 The size of a population at time t years is given by $P(t) = 60\,000\left(1 + 2e^{-\frac{t}{4}}\right)^{-1}$, $t \geqslant 0$.

a Find $P(0)$.

b Find $P'(t)$.

c Show that $P'(t) > 0$ for all $t \geqslant 0$. Explain what this means.

d Find $P''(t)$.

e Find the maximum growth rate of the population, and the exact time when this occurs.

f Discuss $P(t)$ as $t \to \infty$.

g Sketch the population function, showing the information you have found.

56 **a** Find $\int x^2 e^{1-x^3}\, dx$ using the substitution $u(x) = 1 - x^3$.

b Hence show that $\int_0^1 x^2 e^{1-x^3}\, dx = \frac{e-1}{3}$.

57 Suppose $f(x)$ is defined by $f : x \mapsto \cos^3 x$.

a State the range of f.

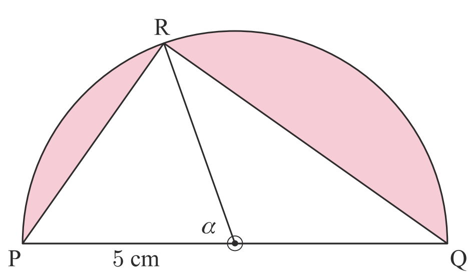
b For the interval $0 \leqslant x \leqslant 2\pi$, how many solutions does $8\cos^3 x = 1$ have?

c Find $f'(x)$.

d $h(x) = \sqrt{3}\cos x\sqrt{\sin x}$ is defined on $0 \leqslant x \leqslant \frac{\pi}{2}$.

When $h(x)$ is revolved about the x-axis through one revolution, a solid is generated. Find the volume of this solid.

58



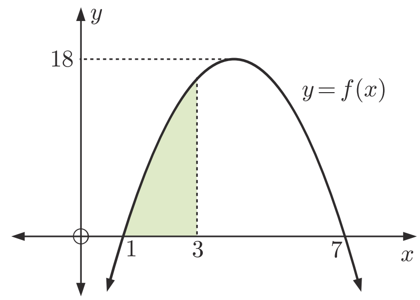
[PQ] is the diameter of a semi-circle with centre O and radius 5 cm.

a Find the area of triangle PQR.

b Hence, find the shaded area A in terms of the angle α.

c Find the maximum and minimum values of the area A, and the values of α when they occur.

59 The graph of the function $f(x) = a(x-h)^2 + k$ is shown alongside. It has x-intercepts 1 and 7, and a maximum value of 18.



a Find the value of:

i h **ii** k **iii** a

b Find the shaded area.

60 **a** Find the exact value of x for which:

i $2^{1-2x} = 0.5$ **ii** $\log_x 7 = 5$

b Solve for x: $25^x - 6(5^x) + 5 = 0$.

c If $2^x = 3^{1-x}$, show that $x = \log_6 3$.

61 **a** Suppose $\dfrac{1-\cos 2\theta}{\sin 2\theta} = \sqrt{3}$ where $0 < \theta < \frac{\pi}{2}$.

i Show that $\tan\theta = \sqrt{3}$ also. **ii** Find θ.

b If $\cos 2x = 2\cos x$, find the value of $\cos x$.

62 The sum of the first n terms of a series is given by $S_n = n^3 + 2n - 1$.
Find u_n, the nth term of the series.

63 Find the *exact* values of x for which $\sin^2 x + \sin x - 2 = 0$ and $-2\pi \leqslant x \leqslant 2\pi$.

64 If $f : x \mapsto \ln x$ and $g : x \mapsto 3 + x$, find:

a $f^{-1}(2) \times g^{-1}(2)$ **b** $(f \circ g)^{-1}(2)$.

65 The equation of line L is $\mathbf{r} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k} + t(-\mathbf{i} + \mathbf{j} - \mathbf{k})$, $t \in \mathbb{R}$. Find the coordinates of the point on L that is nearest to the origin.

66 For the function $f(x)$, $f'(x) > 0$ and $f''(x) < 0$ for all $x \in \mathbb{R}$, $f(2) = 1$, and $f'(2) = 2$.

a Find the equation of the tangent to $f(x)$ where $x = 2$.

b On the same set of axes, sketch $y = f(x)$ and the tangent to the curve where $x = 2$.

c Explain why $f(x)$ has exactly one zero.

d Estimate an interval in which the zero of $f(x)$ lies.

67 In a team of 30 judo players, 13 have won a match by throwing (T), 12 have won by hold-down (H), and 13 have won by points decision (P). 2 have won matches by all three methods. 5 have won matches by throwing and hold-down. 4 have won matches by hold-down and points decision. 3 have won matches by throwing and points decision.

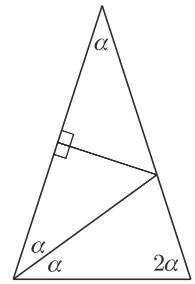
a Draw a Venn diagram to display this information.

b Find:

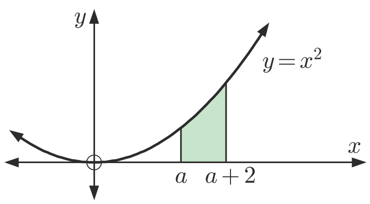
i $P(T \cap H)$ **ii** $P(P)$ **iii** $P(H \cap P')$

iv $P(T \cup P)$ **v** $P(T \cap H' \cap P)$

68 Use the figure alongside to show that $\cos 36° = \frac{1+\sqrt{5}}{4}$.



69



Find a given that the shaded region has area $5\frac{1}{6}$ units^2.

70 What can be deduced if $A \cap B$ and $A \cup B$ are independent events?

71 Solve $\sin\theta\cos\theta = \frac{1}{4}$ on the domain $-\pi \leqslant \theta \leqslant \pi$.

72 f is defined by $x \mapsto \ln(x(x-2))$.

a State the domain of $f(x)$.

b Find $f'(x)$.

c Find the equation of the tangent to $y = f(x)$ at the point where $x = 3$.

73 Hat 1 contains three green and four blue tickets. Hat 2 contains four green and three blue tickets. One ticket is randomly selected from each hat.

a Find the probability that the tickets are the same colour.

b Given that the tickets are different colours, what is the probability that the green ticket came from Hat 2?

74 A normally distributed random variable X has a mean of 90.
The probability $P(X < 85) \approx 0.1587$.

a Find $P(90 < X < 95)$.

b *Estimate* the standard deviation for the random variable X.

75 The discrete random variable X has probability function $P(X = x) = a\left(\frac{2}{5}\right)^x$, $x = 0, 1, 2, 3, ...$
Find the value of a.

76 Given $x = \log_3 y^2$, express $\log_y 81$ in terms of x.

77 Matt has noticed that his pet rat Pug does not always eat the same amount of food each day. He wonders whether this is connected to the temperature, so he decides to collect some data.

Temperature (°C)	10	16	12	19	20
Food eaten (grams)	15	12	13	9	11

a Draw a scatter diagram for this data.

b Hence describe the correlation between *temperature* and *food eaten*.

c Find the mean point for the data.

d Hence draw the line of best fit on your scatter diagram.

e Determine the equation of your line of best fit.

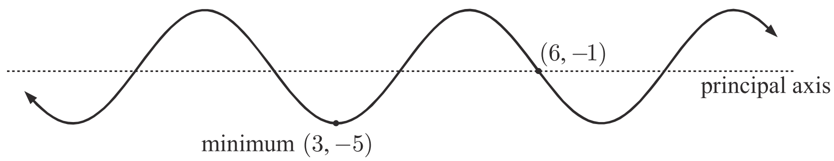
f Hence estimate the amount of food Pug would eat on a 5°C day.

g Comment on the reliability of your prediction.

78 The point $A(-2, 3)$ lies on the graph of $y = f(x)$. Give the coordinates of the point that A moves to under the transformation:

a $y = f(x-2)+1$ **b** $y = 2f(x-2)$ **c** $y = f(2x)-3$ **d** $y = f^{-1}(x)$

79 Find a trigonometric equation in the form $y = a\sin(b(x-c))+d$ which represents the following information:



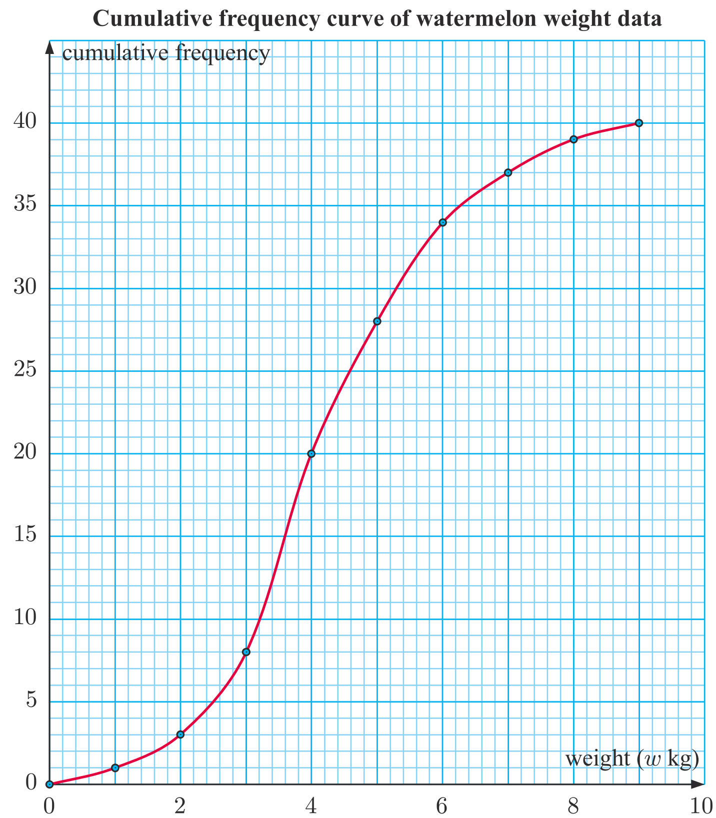
80 A and B are events for which $P(A) = 0.3 + x$, $P(B) = 0.2 + x$, and $P(A \cap B) = x$.

a Find x if A and B are mutually exclusive events.

b Calculate the possible values of x if A and B are independent events.

81 Simplify: **a** $9^{\log_3 11}$ **b** $\log_m n \times \log_n m^2$

82



The graph describes the weight of 40 watermelons.

a Estimate the:

i median weight

ii IQR

for the weight of the watermelons.

b Construct a frequency table corresponding to the data.

c Estimate the mean weight of the watermelons.

83 If $f : x \mapsto 2x+1$ and $g : x \mapsto \dfrac{x+1}{x-2}$, find: **a** $(f \circ g)(x)$ **b** $g^{-1}(x)$.

84 A and B are two events such that $P(A) = \frac{1}{3}$ and $P(B) = \frac{2}{7}$.
Find $P(A \cup B)$ if A and B are: **a** mutually exclusive **b** independent.

85 Find x in terms of a if $a > 1$ and $\log_a(x+2) = \log_a x + 2$.

86 Consider the expansion $(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$.

a Write down the binomial expansion for $(a-b)^5$.

b Simplify $(0.4)^5 + 5(0.4)^4(0.6) + 10(0.4)^3(0.6)^2 + 10(0.4)^2(0.6)^3 + 5(0.4)(0.6)^4 + (0.6)^5$.

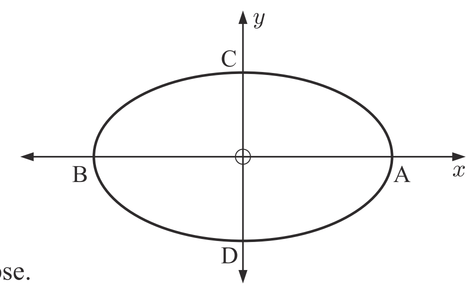
c Write $\left(2x + \frac{1}{x}\right)^5$ in simplified expanded form.

87 If $x + \frac{1}{x} = a$, find in terms of a:

a $x^2 + \frac{1}{x^2}$ **b** $x^3 + \frac{1}{x^3}$

88 The illustrated ellipse has equation

$$\frac{x^2}{16} + \frac{y^2}{4} = 1.$$



a Find the coordinates of points:

i A and B **ii** C and D.

b State the equation of the top half BCA of the ellipse.

c Write a definite integral for the area of the ellipse.

d If the ellipse is rotated through 2π about the x-axis, a solid of revolution is generated. Find the exact volume of this solid.

89 Consider $f(x) = \sin^2 x$.

a Copy and complete the table of values:

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
$f(x)$	0	$\frac{1}{2}$			0			$\frac{1}{2}$	

b Sketch the graph of $f(x) = \sin^2 x$ on the domain $0 \leqslant x \leqslant 2\pi$.

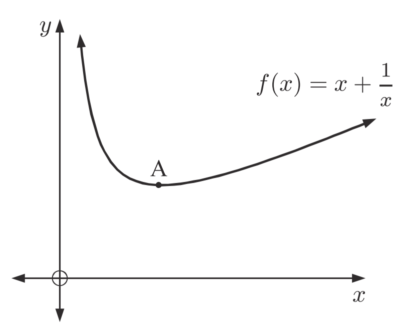
c Check your graph by plotting the point where $x = \frac{\pi}{6}$.

d State the range of $f(x)$.

e Using $\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$, find the area enclosed by $y = f(x)$ and the x-axis for $0 \leqslant x \leqslant \pi$.

f Find the equation of the tangent to $y = f(x)$ at the point $\left(\frac{\pi}{4}, \frac{1}{2}\right)$.

90 The graph of $f(x) = x + \frac{1}{x}$, $x > 0$ is shown.



a Find $f'(x)$ and solve the equation $f'(x) = 0$.

b Find the coordinates of the local minimum A.

c Copy and complete:
"the sum of a positive number and its reciprocal is at least ......"

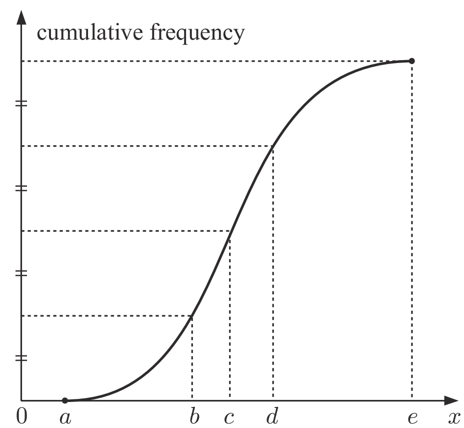
d How many positive solutions would these equations have?

i $x + \frac{1}{x} = 1$ **ii** $x + \frac{1}{x} = 2$ **iii** $x + \frac{1}{x} = 3$ Give reasons for your answers.

91 A straight line passes through A(2, 0, -3) and has direction vector $\mathbf{i}-\mathbf{j}+2\mathbf{k}$.

- **a** Write down the vector equation of the line.
- **b** Write down parametric equations for the line.
- **c** What is represented by $\text{P}(2+t,\ -t,\ -3+2t)$?
- **d** Find $\overrightarrow{\text{BP}}$ given the point B(-1, 3, 5).
- **e** Find $\overrightarrow{\text{BP}} \bullet (\mathbf{i}-\mathbf{j}+2\mathbf{k})$.
- **f** Hence find the value of t when [BP] is perpendicular to the original line.
- **g** What point on the original line is closest to B?

92 A cumulative frequency graph for the continuous random variable X is given alongside.



- **a** What is represented by:
  - **i** a **ii** b **iii** c
  - **iv** d **v** e?
- **b** What do these measure?
  - **i** $e-a$ **ii** $d-b$
- **c** Determine:
  - **i** $\text{P}(b < X \leqslant d)$ **ii** $\text{P}(X > b)$.
- **d** Draw an accurate boxplot for the data set.

B CALCULATOR QUESTIONS

EXERCISE 25B

1 Find n given that $\sum_{k=1}^{n}(2k-31)=0$.

2 Consider the function $f(x)=5\ln(x-4)+2$.

- **a** Graph the function $y=f(x)$. Clearly label the axes intercepts and asymptotes.
- **b** Solve the equation $f(x)=1$.
- **c** Graph the function $y=f^{-1}(x)$ on the same set of axes. Clearly label the axes intercepts and asymptotes.
- **d** Find the equation of the normal to the curve $y=f(x)$ at the point where $x=5$.

3 Find the constant term in the expansion of $\left(x-\dfrac{1}{5x^2}\right)^9$.

4 The value of a cash investment after t years is given by $V=7500\times 2^{0.09t}$ dollars.

- **a** Find the initial value of the cash investment.
- **b** Find the value of the investment after:
  - **i** 5 years **ii** 15 years.
- **c** What was the percentage increase in the investment in the first five years?
- **d** How many years will it take for the investment to double in value?