Calculus Midterm Review III: Derivatives

Name: _____ **Date:** _____

1. Find all critical numbers of $f(x) = 3x^4 - 2x^3$.

   A. 1 only
   B. 0 and $\frac{1}{2}$
   C. 0, 1, 6
   D. 0, $\frac{1}{2}$, 1

2. If $f(x) = x\sqrt{2x+1}$, find all critical numbers.

   A. 0, $-\frac{1}{3}$, and $-\frac{1}{2}$
   B. $\frac{1}{2}$ and $\frac{1}{3}$
   C. 0 and $-\frac{1}{2}$
   D. $-\frac{1}{3}$ only

3. Given that $f(x) = -x^2 + 12x - 28$ has a relative maximum at $x = 6$, choose the correct statement.

   A. f' is negative on the interval $(-\infty, 6)$
   B. f' is positive on the interval $(-\infty, \infty)$
   C. f' is negative on the interval $(6, \infty)$
   D. f' is negative for all real values

4. Let f be defined by $f(x) = (x^2 - 1)^3$ for all real numbers x. Find the x and y coordinates of the relative maximum and minimum points.

   A. $(1, 0)$ max
   B. $(-1, 0)$ min
   C. $(0, -1)$ min
   D. $(0, -1)$ max, $(-1, 0)$ min

5. Find the absolute maximum value of $f(x) = x^3 - 3x^2 - 9x$ on the closed interval $[0, 6]$.

   A. -27 B. 27 C. 54 D. 216

6. Which of the following gives the relative extrema for the function $f(x) = (x-3)^2(x+4)$?

   A. Relative maximum: $x = -\frac{5}{3}$; Relative minimum: $x = 3$
   B. Relative maxima: $x = \frac{5}{3}$, $x = 3$; Relative minimum: $x = -3$
   C. Relative maximum: $x = \frac{5}{3}$; Relative minimum: $x = -3$
   D. Relative maximum: $x = -\frac{3}{5}, 3$; Relative minimum: $x = 3$

7. Given the curve $f(x) = x^2e^x$, find the y-coordinate of the relative maximum point.

   A. -2 B. 2 C. $\frac{4}{e^2}$ D. $\frac{2}{e^4}$

8. How many critical points does f have on the open interval $(0, 10)$ if $f'(x) = \frac{e^{-2x}}{x^3} - \sin x$?

   A. 2 B. 3 C. 4 D. 5

9. If $f(x) = \cos^4 x$, then $f'(\frac{\pi}{3}) =$

   A. $-\frac{\sqrt{3}}{4}$ B. $\frac{1}{2}$ C. $\frac{4}{\sqrt{3}}$ D. $\frac{\sqrt{3}}{2}$

10. If $f(x) = \sin(2x)\cos x$, then $f'(\frac{\pi}{3}) =$

    A. $\sqrt{3} + 1$
    B. $\frac{5}{4}$
    C. $-\frac{5}{4}$
    D. $\frac{\pi}{3}$

11. Given $f(x) = e^{\sqrt{2x}}$, find $f'(2)$.

    A. e^2 B. $2e$ C. $\frac{e^2}{4}$ D. $\frac{e^2}{2}$

12. Find the derivative of $x^2 f(x)$.

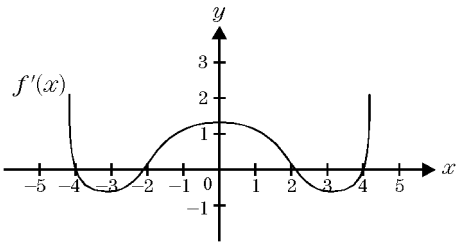
A. $x\left[xf'(x) + 2f(x)\right]$ B. $x\left[xf(x) + 2f'(x)\right]$

C. $x^2 f'(x)$ D. $\frac{1}{3}x^3 + \left[f'(x)\right]^2$

13. Assume $f(7) = 0$, $f'(7) = 14$, $g(7) = 1$, and $g'(7) = \frac{1}{7}$. Find $h'(7)$ given $h(x) = \dfrac{f(x)}{g(x)}$.

A. -14 B. 14 C. $\frac{49}{2}$ D. 98

14. The graph $f(x)$ has horizontal tangents when $x =$



A. $-3,\ 0,\ 3$ B. $-4,\ 2$

C. $-4,\ -2,\ 2,\ 4$ D. $2,\ 4$

15. Find an equation of the tangent line to the graph of $f(x) = \frac{(x-3)}{(x+3)}$ when $x = -2$.

A. $y - 5 = 6(x + 2)$ B. $y + 5 = 6(x + 2)$

C. $y - 5 = -6(x - 2)$ D. $y - 5 = 3(x + 2)$

16. The graph of $f(x) = \dfrac{3x^2}{16 - x^2}$ has a horizontal tangent at $y =$

A. -3 B. 3 C. 4 D. 0

17. Given the curve

$$f(x) = \ln x - x + 1 + \frac{(x-1)^2}{2}$$

then a horizontal tangent exists when $x =$

A. $\ln(e^2)$ B. e^{-3} C. $\ln 1$ D. $\ln e$

18. Given a function defined by $y - 3 = \sqrt{16 - 9x^2}$. For what points on the curve does the function have one or more vertical tangents?

A. $(0, 7)$ and $(0, -7)$

B. $(-\frac{4}{3}, 3)$ and $(\frac{4}{3}, 3)$

C. $(\frac{16}{3}, 3)$ and $(-\frac{16}{9}, 3)$

D. $(-\frac{4}{3}, -3)$ and $(\frac{4}{3}, -3)$

19. If $f(x) = x \sin x$, determine the equation of the tangent line to the graph when $x = \dfrac{\pi}{2}$.

A. $y = 0$ B. $f'(x) = 0$

C. $y = x\cos x + \sin x$ D. $y = x$

20. Given $y^3 = x^3 - 1$, find y''.

A. $2xy^{-2}(1 - x^3y^{-3})$ B. $2xy^2\left(1 - \dfrac{x^3}{y^3}\right)$

C. $\dfrac{2x^2 - 2x^2y}{y^4}$ D. $\dfrac{2xy^2 - x^2y}{y^4}$

21. If $y = 2\ln 3x$, then $\dfrac{d^2y}{dx^2} =$

A. $\dfrac{2}{x^2}$ B. $-\dfrac{2}{x^2}$ C. $\dfrac{4}{9x^2}$ D. $-\dfrac{4}{9x^2}$

22. Suppose $f(x) = x^3$ and let $h(x)$ be the inverse of f. Find $h'(-8)$.

A. $\frac{1}{12}$ B. $-\frac{1}{6}$ C. $\frac{1}{6}$ D. $-\frac{1}{12}$

23. $f(x)$ and $h(x)$ are inverse functions. If $f(x) = x^2 - x - 15$, then find $h'(5)$ for $x > 0$.

A. $\frac{1}{9}$ B. $-\frac{1}{9}$ C. 12 D. $-\frac{1}{8}$

24. Find $f'(x)$ given $f(x) = \sin^3(4x)$.

A. $3\sin^2 4x\cos(4x)$ B. $\cos^3 4x$

C. $12\sin^2 4x\cos(4x)$ D. $12\cos^2(4x)$

25. If $x = y + 3y^2 + 4y^3$, then $y' =$

A. $\dfrac{1}{1+6y+12y^2}$ B. $\dfrac{4}{3(1+6y+8y^2)}$

C. $\dfrac{1}{6y+12y^2-x}$ D. $1+6y+12y^2$

26. If $y = \dfrac{y+x}{xy}$, then $\dfrac{dy}{dx} =$

A. $\dfrac{y^2}{2xy}$ B. $\dfrac{y^2-1}{1-2xy}$

C. $\dfrac{y^2}{x^2}$ D. $\dfrac{y^2+1}{1-2xy}$

27. What is the slope of the tangent line to $xy + \ln 2x = \frac{1}{2}$ at the point $(\frac{1}{2}, 1)$?

A. -6 B. $-\frac{3}{2}$ C. $\frac{2}{3}$ D. $-\frac{1}{6}$

28. Find the acceleration for $x(t) = \sqrt{5t-6}$ at $t = 3$.

A. $-\frac{25}{108}$ B. $-\frac{25}{216}$ C. $\frac{5}{6}$ D. $\frac{5}{12}$

29. How fast is the area of a square increasing when the side is 5 m in length and growing at a rate of 0.6 m/min?

A. $7.2\,\text{m}^2/\text{min}$ B. $1.2\,\text{m}^2/\text{min}$

C. $3.0\,\text{m}^2/\text{min}$ D. $6.0\,\text{m}^2/\text{min}$

30. A stone dropped in a still pond creates a circular ripple whose radius increases at a constant rate of 3 ft/s. At what rate is the area enclosed by the ripple increasing 8 s after the stone strikes the pond?

A. $48\pi\,\text{ft}^2/\text{s}$ B. $64\pi\,\text{ft}^2/\text{s}$

C. $128\pi\,\text{ft}^2/\text{s}$ D. $144\pi\,\text{ft}^2/\text{s}$

31. A man 2 m tall walks away from a lamppost whose light is 5 m above the ground. If he walks at a speed of 1.4 m/s, at what rate is his shadow growing when he is 10 m from the lamppost?

A. $\frac{14}{15}$ m/s B. $\frac{7}{5}$ m/s

C. $\frac{5}{7}$ m/s D. $\frac{2}{5}$ m/s

32. Two vehicles are approaching an intersection. One truck from the west at 15 m/s and one van from the north at 20 m/s. How fast is the distance between the vehicles changing at the instant the truck is 60 m west and the van 80 m north of the intersection?

A. 10 m/s B. 17 m/s

C. 20 m/s D. 25 m/s

33. A machine is rolling a metal cylinder under pressure. The radius of the cylinder is decreasing at a constant rate of 0.05 inches per second and the volume V is 128π cubic inches. At what rate is the length h changing when the radius r is 1.5 inches? Note: $V = \pi r^2 h$

A. -75.853 in/sec B. 3.793 in/sec

C. -3.793 in/sec D. 9.481 in/sec

34. Sand is falling of a conveyor onto a conical pile at the rate of 15 feet^3 per minute. The diameter of the base of the cone is twice the altitude. At what rate is the height of the pile changing when it is 10 feet high?

A. $\frac{3}{20\pi}$ ft/min B. $\frac{20}{3\pi}$ ft/min

C. $\frac{20\pi}{3}$ ft/min D. $\frac{3}{20}$ ft/min

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1.
Answer: B

2.
Answer: D

3.
Answer: C

4.
Answer: C

5.
Answer: C

6.
Answer: A

7.
Answer: C

8.
Answer: C

9.
Answer: A

10.
Answer: C

11.
Answer: D

12.
Answer: A

13.
Answer: B

14.
Answer: C

15.
Answer: B

16.
Answer: D

17.
Answer: D

18.
Answer: B

19.
Answer: D

20.
Answer: A

21.
Answer: B

22.
Answer: A

23.
Answer: A

24.
Answer: C

25.
Answer: A

26.
Answer: B

27.
Answer: A

28.
Answer: A

29.
Answer: D

30.
Answer: D

31.
Answer: A

32.
Answer: D

33.
Answer: B

34.
Answer: A