

**Calculus Midterm Review II: Derivatives**

Name: \_\_\_\_\_

Date: \_\_\_\_\_

1.  $f$  and  $g$  are differentiable functions and have the values shown in the table.

If  $A = 5f - 8g$ , then  $A'(0) =$

- A. 0  
B. 3  
C. 61  
D. 7

$x$	$f$	$f'$	$g$	$g'$
0	5	1	-7	$\frac{1}{4}$
2	8	3	-5	1
4	14	9	-3	4
6	26	27	-1	16

2. The functions  $f$  and  $g$  are differentiable and have the values shown in the table.

If  $A = f \cdot g$ , then  $A'(4) =$

- A. 0      B. 114      C. 83      D. 29

3. The functions  $f$  and  $g$  are differentiable and have the values shown in the table.

If  $A = \left(\frac{1}{g}\right)$ , then  $A'(4) =$

- A. 0      B.  $\frac{4}{9}$       C.  $-\frac{4}{9}$       D.  $-\frac{1}{4}$

4. The functions  $f$  and  $g$  have the values shown in the table and are differentiable.

If  $A = \left(\frac{f}{g}\right)$ , then  $A'(2) =$

- A.  $-\frac{23}{4}$       B.  $\frac{23}{4}$       C. -7      D.  $-\frac{23}{25}$

5. The functions  $f$  and  $g$  are differentiable and have the values shown in the table.

If  $A = f(g(x))$ , then  $A'(-8) =$

- A. -72  
B. 54  
C. 9  
D. -9

$x$	$f$	$f'$	$g$	$g'$
-8	4	3	-2	6
-6	10	12	0	9
-2	20	9	6	18
2	30	15	12	24

6.  $f$  and  $g$  are differentiable functions and have the values shown in the table.

If  $A = \sqrt{f(x)}$ , then  $A'(-2) =$

- A.  $\frac{9}{8}$   
B. impossible  
C.  $\frac{8}{9}$   
D. 3

$x$	$f$	$f'$	$g$	$g'$
-8	4	3	-2	6
-6	10	12	0	9
-2	16	9	6	18
2	30	11	12	24

7. The functions  $f$  and  $g$  are differentiable and have the values shown in the table.

If  $A = f(x^3)$ , then  $A'(-2) =$

- A. -18  
B. 9  
C. -120  
D. 36

$x$	$f$	$f'$	$g$	$g'$
-8	4	3	-2	6
-6	10	12	0	9
-2	16	9	6	18
2	30	15	12	24

8. Given functions  $f$  and  $g$ , which are differentiable and have the values shown in the table.

If  $A = f^{-1}$ , then  $A'(4) =$

- A. 12  
B. 3  
C.  $\frac{1}{12}$   
D.  $\frac{1}{3}$

$x$	$f$	$f'$	$g$	$g'$
2	4	3	8	6
4	10	12	6	9
6	16	26	4	18
8	30	40	2	24

9. A twice differentiable function  $f$  is given by the table.

Estimate  $f''(4.6)$ :

$x$	3.7	4.3	4.9	5.5	6.1
$f(x)$	1.8	3.4	4.6	6.4	8.4

- A. 4.900      B. 0.278      C. 3.593      D. 2.500

10. The table shows the position of an object moving along a line at 10 second intervals.

Estimate the velocity, in units/sec, at  $t = 15$ .

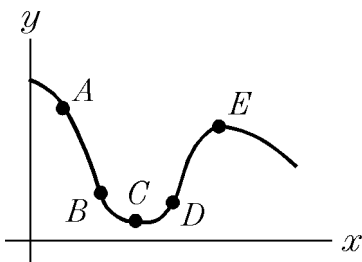
$t$ (sec)	0	10	20	30	40
position	4	12	26	44	68

- A. 0.140                      B. -0.714  
 C. 0.714                      D. 1.400
11. The table shows the velocity of an object moving along a line at 2 second intervals.

Estimate the acceleration, in units/sec<sup>2</sup>, at  $t = 10$ .

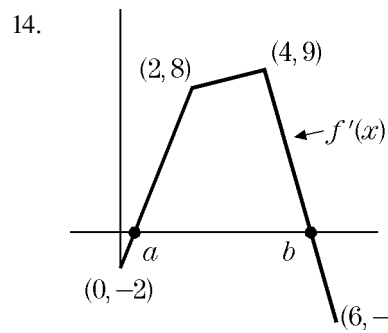
$t$ (sec)	10	12	14	16	18
$v$ (units/sec)	18.0	22.8	27.6	33.4	42.2

- A. 1.800                      B. 0.417  
 C. -0.417                      D. 2.400
12. At which of the five points shown on the graph are  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$  both positive?



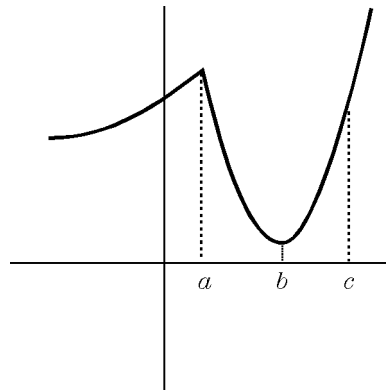
- A. B                      B. C                      C. D                      D. E
13. At which of the five points shown on the graph is  $\frac{d^2y}{dx^2}$  negative? Choose the *best* answer.

- A. A and E                      B. C only  
 C. B, C, and D                      D. E only



The graph of  $f'$  is shown. If  $f(2) = 17$ , then  $f(0) =$  \_\_\_\_\_.

- A. 1                      B. -1                      C. -33                      D. 10
15. Which of the following tables goes best with the graph of  $f$ ?



- A. 

$x$	$f'(x)$
$a$	0
$b$	0
$c$	4
- B. 

$x$	$f'(x)$
$a$	0
$b$	0
$c$	-2
- C. 

$x$	$f'(x)$
$a$	does not exist
$b$	0
$c$	6.2
- D. 

$x$	$f'(x)$
$a$	does not exist
$b$	does not exist
$c$	-1

16. Let  $f$  be defined by  $f(x) = (x^2 - 1)^4$  for all real numbers  $x$ . For what values of  $x$  is the function increasing?
- A.  $(-1, 1)$   
 B.  $(-1, 0)$   
 C.  $(-1, 0)$  and  $(1, \infty)$   
 D.  $(-\infty, -1)$  and  $(1, \infty)$
17. Let  $f(x) = x^2(x - 3)$ . Over what interval is the function decreasing?
- A.  $0 < x < 2$   
 B.  $0 < x < \infty$   
 C.  $-\infty < x < \infty$   
 D.  $-\infty < x < 0$  and  $x > 2$
18. If  $f(x) = \frac{1}{2} \sin 2x$  for  $0 \leq x < 2\pi$ , determine the interval(s) over which the function is increasing.
- A.  $0 < x < \frac{\pi}{2}$  and  $\frac{3\pi}{2} < x < \frac{\pi}{2}$   
 B.  $0 < x < \frac{\pi}{4}$ ,  $\frac{3\pi}{4} < x < \frac{5\pi}{4}$  and  $\frac{7\pi}{4} < x < 2\pi$   
 C.  $\frac{\pi}{4} < x < \frac{3\pi}{4}$   
 D.  $0 < x < \frac{\pi}{4}$
19. Given that  $f'(x) < 0$  for all  $x$  in the domain of  $f$ .  
 What can be said about the number of solutions of  $f(x) = 10$ ?
- A. 0  
 B. 1  
 C. 0 or 1  
 D. not enough information
20. Show that  $f(x) = x^2 - 3x + 2$  satisfies Rolle's Theorem on  $[1, 2]$  and find the corresponding value of  $c$  that the theorem guarantees.
21. Why does Rolle's Theorem not apply to  $f(x) = |x - 3|$  on the interval  $[-1, 7]$ ?
22. Given  $f(x) = \ln(x - 2) + \sin x$ . Find the first non-negative interval,  $[a, b]$ , for which Rolle's Theorem applies and find the corresponding value of  $c$ .
23. Given the function  $f(x) = x(x^2 - 8) - 5$  satisfies the hypothesis of the Mean Value Theorem on the interval  $[1, 4]$ , find a number  $C$  in the interval  $(1, 4)$  which satisfies this theorem.
- A.  $\sqrt{5}$     B.  $\sqrt{7}$     C. 12    D. 5
24. Given  $f(x) = \sqrt[3]{x}$  on  $[-1, 5]$ . Why does  $f$  not satisfy the hypothesis of the Mean Value Theorem?
- A.  $f'(-1)$  does not exist  
 B.  $f'(0)$  does not exist  
 C.  $f$  is not continuous on  $[-1, 5]$   
 D.  $f$  is not differentiable for  $x < 0$
25. What is the tangent line approximation to  $y = x^3 - 7x + 3$  for values of  $x$  near 2?
- A.  $y = -3 - 5(x + 2)$     B.  $y = -3 + 5(x - 2)$   
 C.  $y = -3 - 5(x - 2)$     D.  $y = 3 - 5(x + 2)$
26. What is the average rate of change over  $3 \leq t \leq 7$ ?
- |        |     |     |     |      |      |
|--------|-----|-----|-----|------|------|
| $t$    | 3   | 5   | 7   | 9    | 11   |
| $f(t)$ | 1.2 | 4.6 | 8.2 | 13.4 | 20.4 |
- A. 3.5    B. 1.75    C. 0.286    D. -1.75
27. The position of an object is given by  $s = t^2 - 3t + 8$ . What is its average velocity for  $3 \leq t \leq 5$ ?
- A. 4    B. 3.333    C. 5    D. 0.2

28. Find all points of inflection of the function  $f(x) = x^4 - x^3$ .

- A. (0, 0)  
B. (0, 0) and  $(\frac{1}{2}, -\frac{1}{16})$   
C.  $(\frac{1}{2}, -\frac{1}{16})$   
D.  $(\frac{1}{2}, -\frac{1}{16})$  and  $(\frac{3}{4}, -\frac{27}{256})$

29. Let  $f$  be defined by  $f(x) = x + 1 - (\cos x)^2$ . Find the  $x$ -coordinates of all inflection points of  $f$ ,  $\frac{\pi}{6} \leq x \leq \frac{5\pi}{6}$ .

- A.  $\frac{\pi}{4}$     B.  $\frac{\pi}{4}, \frac{3\pi}{4}$     C.  $\frac{\pi}{2}, \frac{3\pi}{4}$     D.  $\frac{\pi}{2}, \frac{5\pi}{6}$

30. Find all intervals on which the function  $y = 6x^3 - x^4$  is concave upward.

- A.  $(-\infty, 0)$  and  $(3, \infty)$     B.  $(-\infty, 0)$  and  $(6, \infty)$   
C. (0, 3)    D. (3, 6)

31. Find all intervals on which the function is concave downward:  $f(x) = \frac{x^2 - 4}{x^2}$ .

- A.  $(-\infty, \infty)$   
B.  $(-\infty, -2)$  and  $(2, \infty)$   
C.  $(-\infty, 0)$  and  $(0, \infty)$   
D.  $(-\infty, -2)$

32. Let  $f''(x) = 4x^3 - 2x$  and let  $f(x)$  have critical numbers  $-1, 0$ , and  $1$ . Use the Second Derivative Test to determine if any of the critical numbers gives a relative maximum.

- A.  $-1$     B.  $1$     C.  $\pm 1$     D.  $0, \pm 1$

33. If  $y = e^{x^2 - 3x}$ , then  $y' =$

- A.  $(2x - 3)e^{x^2 - 3x}$     B.  $2x - 3$   
C.  $(x^2 - 3x)e^{x^2 - 3x}$     D.  $(2x - 3)e^{2x - 3}$

34. If  $y = e^{1/x}$ , then  $y' =$

- A.  $-\frac{e^{1/x}}{x^2}$     B.  $e^{1/x}$     C.  $\frac{e^{1/x}}{x}$     D.  $xe^{1/x}$

35. Find  $y'$  given  $y = e^{\sin \sqrt{x}}$ .

- A.  $\frac{\cos \sqrt{x}}{2\sqrt{x}} e^{\sin \sqrt{x}}$     B.  $(\cos \sqrt{x})e^{\sin \sqrt{x}}$   
C.  $(\sin \sqrt{x})e^{\sin \sqrt{x} - 1}$     D.  $\frac{(\sin \sqrt{x})e^{\sin \sqrt{x}}}{\sqrt{x}}$

36. If  $y = \ln(x^2 - x)$ , then  $\frac{dy}{dx} =$

- A.  $\frac{2x - 1}{x(x - 1)}$     B.  $\frac{2}{x}$   
C.  $\frac{1}{x(x - 1)}$     D.  $\frac{x(x - 1)}{2x - 1}$

37. If  $y = \ln(e^{3x} - 5)$ , then  $\frac{dy}{dx} =$

- A.  $\frac{1}{e^{3x} - 5}$     B.  $\frac{3x - 5}{e^{3x} - 5}$   
C.  $\frac{3e^{3x}}{e^{3x} - 5}$     D.  $e^{3x} - 5$

38. Find  $y'$  given  $3xe^y - 4 = x^2y$ .

- A. 0    B.  $\frac{2xy - 3e^y}{3xe^y - x^2}$   
C.  $\frac{e^y}{3xe^y - 4}$     D.  $\ln 3x$

39. Find  $y'$  given  $xe^y + 1 = xy$ .

- A. 0    B.  $\frac{y - e^y}{xe^y - x}$   
C.  $\frac{y}{e^y - x}$     D.  $\ln x$

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- |                              |   |
|------------------------------|---|
| 1.<br>Answer:      B         | 21.<br>Answer:      not differentiable at $x = 3$ |
| 2.<br>Answer:      D         | 22.<br>Answer: $[2.594, 3.703], c \approx 3.009$  |
| 3.<br>Answer:      C         | 23.<br>Answer:      B                             |
| 4.<br>Answer:      D         | 24.<br>Answer:      B                             |
| 5.<br>Answer:      B         | 25.<br>Answer:      B                             |
| 6.<br>Answer:      A         | 26.<br>Answer:      B                             |
| 7.<br>Answer:      D         | 27.<br>Answer:      C                             |
| 8.<br>Answer:      D         | 28.<br>Answer:      B                             |
| 9.<br>Answer:      B         | 29.<br>Answer:      B                             |
| 10.<br>Answer:      D        | 30.<br>Answer:      C                             |
| 11.<br>Answer:      D        | 31.<br>Answer:      C                             |
| 12.<br>Answer:      C        | 32.<br>Answer:      A                             |
| 13.<br>Answer:      A        | 33.<br>Answer:      A                             |
| 14.<br>Answer:      A        | 34.<br>Answer:      A                             |
| 15.<br>Answer:      C        | 35.<br>Answer:      A                             |
| 16.<br>Answer:      C        | 36.<br>Answer:      A                             |
| 17.<br>Answer:      A        | 37.<br>Answer:      C                             |
| 18.<br>Answer:      B        | 38.<br>Answer:      B                             |
| 19.<br>Answer:      C        | 39.<br>Answer:      B                             |
| 20.<br>Answer: $\frac{3}{2}$ |   |