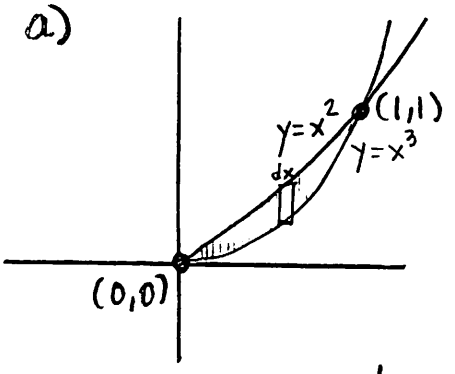
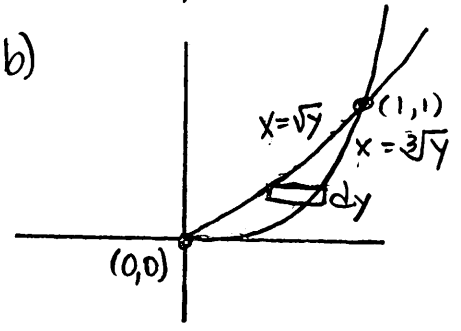
40. a)



$$A = \int_{x=0}^{x=1} (x^2 - x^3)\,dx$$

$$= \left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 = \frac{1}{3} - \frac{1}{4} = \boxed{\frac{1}{12}}$$

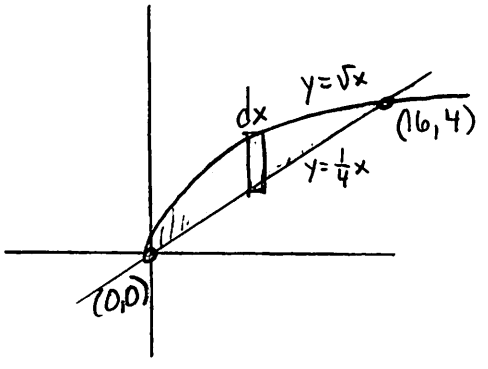
b)



$$A = \int_{y=0}^{y=1} (\sqrt[3]{y} - \sqrt{y})\,dy$$

$$= \left[\frac{3}{4}y^{\frac{4}{3}} - \frac{2}{3}y^{\frac{3}{2}}\right]_0^1 = \frac{3}{4} - \frac{2}{3} = \frac{9-8}{12} = \boxed{\frac{1}{12}}$$

41. a)



$$\sqrt{x} = \frac{1}{4}x$$

$$x = \frac{1}{16}x^2$$

$$16x = x^2$$

$$x^2 - 16x = 0$$

$$x(x-16) = 0$$

$x=0$ or $x=16$

$y=0$ $\quad y=4$

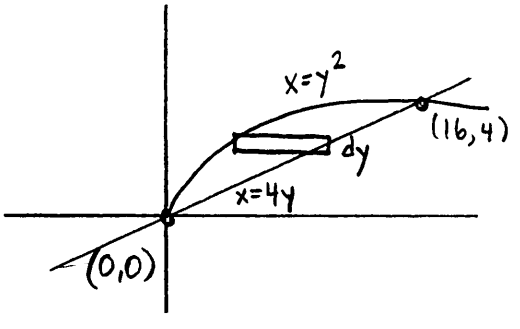
$$A = \int_{x=0}^{x=16} (\sqrt{x} - \frac{1}{4}x)\,dx$$

$$= \left[\frac{2}{3}x^{\frac{3}{2}} - \frac{x^2}{8}\right]_0^{16}$$

$$= \frac{2}{3}(64) - 32$$

$$= \boxed{\frac{32}{3}}$$

b)

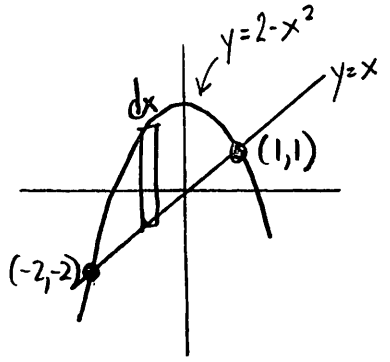


$$A = \int_{y=0}^{y=4} (4y - y^2)\,dy$$

$$= \left[2y^2 - \frac{y^3}{3}\right]_0^4$$

$$= 32 - \frac{64}{3} = \frac{96-64}{3} = \boxed{\frac{32}{3}}$$

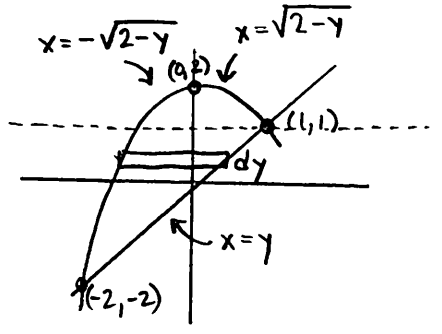
42.



$$A=\int_{x=-2}^{x=1}[(2-x^2)-x]\,dx$$

$$=\left[2x-\frac{x^3}{3}-\frac{x^2}{2}\right]_{-2}^{1}=\left(2-\frac{1}{3}-\frac{1}{2}\right)-\left(-4+\frac{8}{3}-2\right)$$

$$=\boxed{\frac{9}{2}}$$

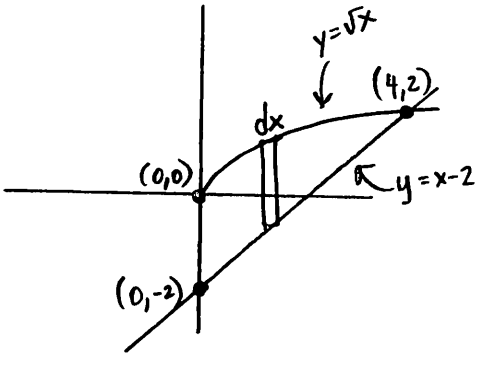


* HORIZONTAL CROSS SECTION IS NOT A GOOD CHOICE FOR THIS TYPE OF PROBLEM

$$A=\int_{y=-2}^{y=1}[y-(-\sqrt{2-y})]\,dy+\int_{y=1}^{y=2}[\sqrt{2-y}-(-\sqrt{2-y})]\,dy$$

$$=3.1\overline{6}+1.3\overline{3}=4.4\overline{9}=\boxed{4.5}$$

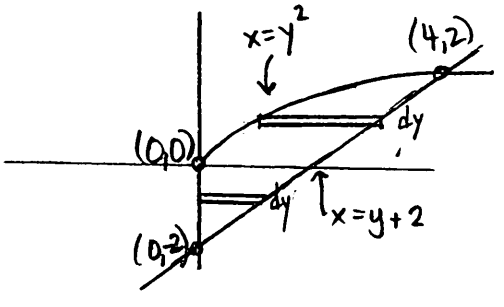
43.



$$A=\int_{x=0}^{x=4}[\sqrt{x}-(x-2)]\,dx$$

$$=\left[\frac{2}{3}x^{\frac{3}{2}}-\frac{x^2}{2}+2x\right]_0^4$$

$$=\frac{16}{3}-8+8=\boxed{\frac{16}{3}}$$



* HORIZONTAL CROSS SECTION IS NOT A GOOD CHOICE FOR THIS TYPE OF PROBLEM

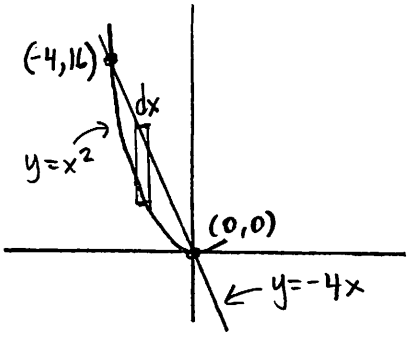
$$A=\int_{y=-2}^{y=0}[(y+2)-0]\,dy+\int_{y=0}^{y=2}[(y+2)-y^2]\,dy$$

$$=\left[\frac{y^2}{2}+2y\right]_{-2}^{0}+\left[\frac{y^2}{2}+2y-\frac{y^3}{3}\right]_0^2$$

$$=(0+0)-(2-4)+\left(2+4-\frac{8}{3}\right)-(0+0-0)$$

$$=2+6-\frac{8}{3}=\frac{24-8}{3}=\boxed{\frac{16}{3}}$$

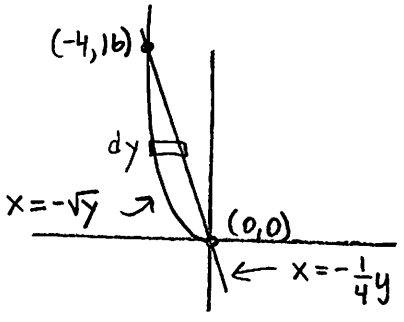
44.



$$A=\int_{x=-4}^{x=0}(-4x-x^2)\,dx$$

$$=\left[-2x^2-\frac{x^3}{3}\right]_{-4}^{0}=(0-0)-\left(-32+\frac{64}{3}\right)$$

$$=\boxed{\frac{32}{3}}$$

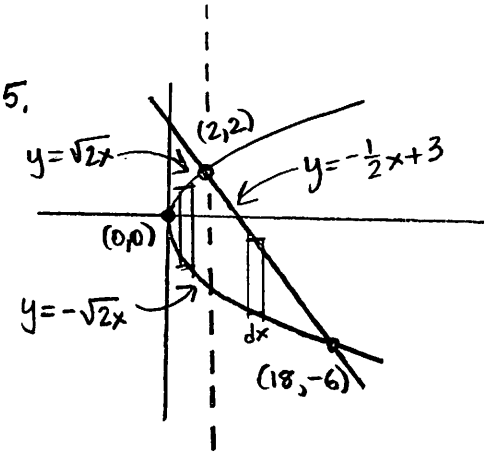


$$A=\int_{y=0}^{y=16}\left[-\frac{1}{4}y-(-\sqrt{y})\right]dy$$

$$=\int_0^{16}\left(-\frac{1}{4}y+\sqrt{y}\right)dy$$

$$=\left[-\frac{y^2}{8}+\frac{2}{3}y^{\frac{3}{2}}\right]_0^{16}=\left(-32+\frac{128}{3}\right)=\boxed{\frac{32}{3}}$$

45.

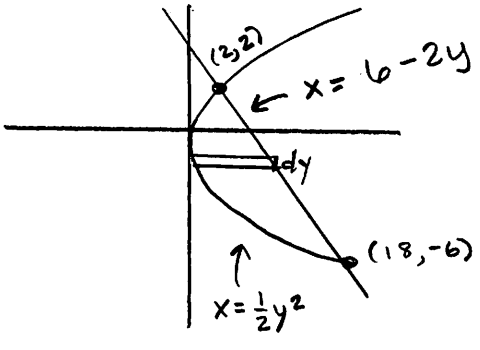


* VERTICAL CROSS SECTION IS NOT A GOOD CHOICE FOR THIS TYPE OF PROBLEM

$$A=\int_{x=0}^{x=2}\left[\sqrt{2x}-(-\sqrt{2x})\right]dx+\int_{x=2}^{x=18}\left[\left(-\frac{1}{2}x+3\right)-(-\sqrt{2x})\right]dx$$

$$=\int_0^2 2\sqrt{2x}\,dx+\int_2^{18}\left(-\frac{1}{2}x+3+\sqrt{2x}\right)dx$$

$$=5.\bar{3}+37.\bar{3}=\boxed{\frac{128}{3}}$$



$$A=\int_{y=-6}^{y=2}\left[(6-2y)-\frac{1}{2}y^2\right]dy$$

$$=\left[6y-y^2-\frac{y^3}{6}\right]_{-6}^{2}$$

$$=\left(12-4-\frac{8}{6}\right)-(-36-36+36)$$

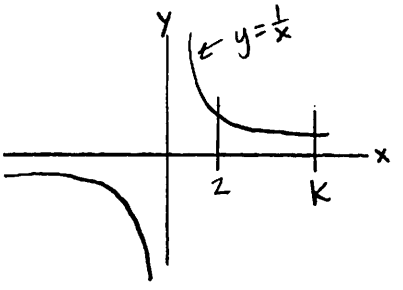
$$=\frac{20}{3}+36=\boxed{\frac{128}{3}}$$

46. $f(x) = g(x) + 5 \quad \therefore \quad f(x) > g(x)$

$$A = \int_{x=a}^{x=b} [f(x) - g(x)]\,dx = \int_a^b 5\,dx = [5x]_a^b = \boxed{5b - 5a}$$

* assuming $g(x)$ is continuous on $[a,b]$ and differentiable on (a,b)

47.

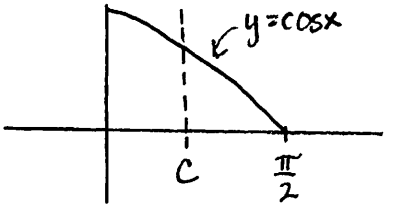


$$A = \int_{x=2}^{x=k} \frac{1}{x}\,dx = [\ln|x|]_2^K$$

$$= \ln|k| - \ln 2$$

$$= \ln\left|\frac{K}{2}\right| = \ln 4 \quad \therefore \boxed{k = 8}$$

48.



$$\int_{x=0}^{x=C} \cos x\,dx = \int_{x=c}^{x=\frac{\pi}{2}} \cos x\,dx$$

$$[\sin x]_0^C = [\sin x]_C^{\frac{\pi}{2}}$$

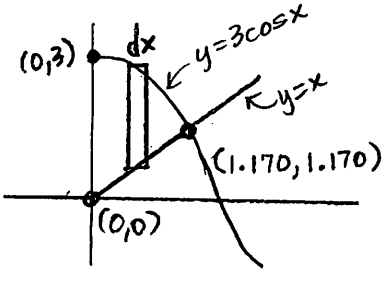
$$\sin c - \sin 0 = \sin\frac{\pi}{2} - \sin c$$

$$2\sin c = 1$$

$$\sin c = \frac{1}{2}$$

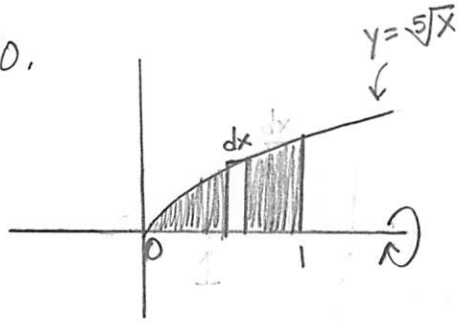
$$\boxed{c = \frac{\pi}{6}}$$

49.



$$A = \int_{x=0}^{x=1.17} (3\cos x - x)\,dx = \boxed{2.078}$$

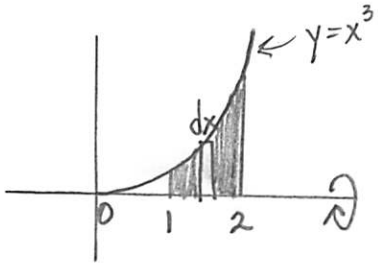
50.



$$V = \pi \int_{x=0}^{x=1} (\sqrt[5]{x})^2 dx = \pi \int_0^1 x^{2/5} dx$$

$$= \pi \left[\frac{5}{7} x^{7/5} \right]_0^1 = \pi \left[\frac{5}{7} - 0 \right] = \boxed{\frac{5\pi}{7}}$$

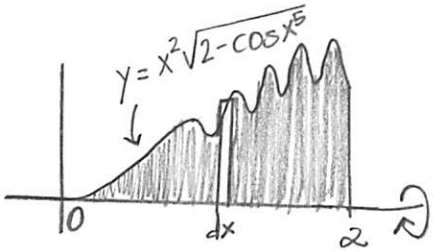
51.



$$V = \pi \int_{x=1}^{x=2} (x^3)^2 dx = \pi \int_1^2 x^6 dx$$

$$= \pi \left[\frac{x^7}{7} \right]_1^2 = \pi \left[\frac{128}{7} - \frac{1}{7} \right] = \boxed{\frac{127\pi}{7}}$$

52.



$$V = \pi \int_{x=0}^{x=2} \left[x^2 \sqrt{2 - \cos(x^5)} \right]^2 dx$$

$$= \pi \int_0^2 x^4 (2 - \cos(x^5)) dx$$

$$u = x^5$$

$$\frac{du}{dx} = 5x^4$$

$$\frac{1}{5} du = x^4 dx$$

$$= \pi \int_a^b \frac{1}{5} (2 - \cos u) du$$

$$= \frac{\pi}{5} \left[2u - \sin u \right]_a^b$$

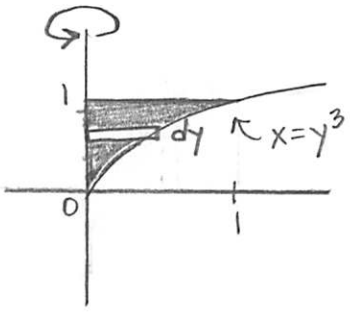
$$= \frac{\pi}{5} \left[2x^5 - \sin(x^5) \right]_0^2$$

$$= \frac{\pi}{5} \left[(64 - \sin 32) - (0 - 0) \right]$$

$$= \frac{64\pi}{5} - \frac{\pi \sin(32)}{5}$$

$$= \boxed{39.866}$$

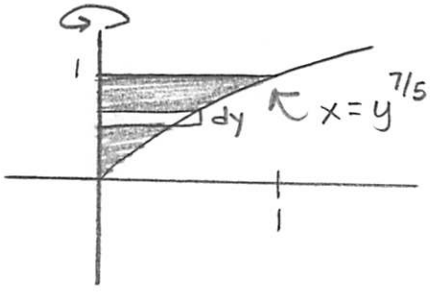
53.



$$V=\pi\int_{y=0}^{y=1}(y^3)^2\,dy=\pi\int_0^1 y^6\,dy$$

$$=\pi\left[\frac{y^7}{7}\right]_0^1=\boxed{\frac{\pi}{7}}$$

54.



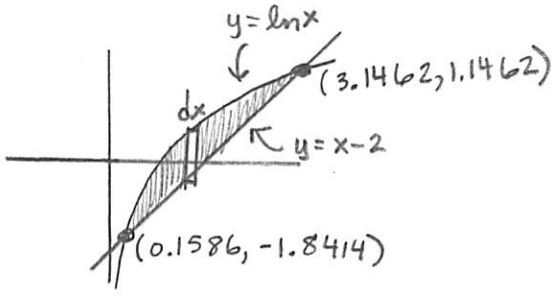
$y=x^{\frac{5}{7}}$

$y^{\frac{7}{5}}=x$

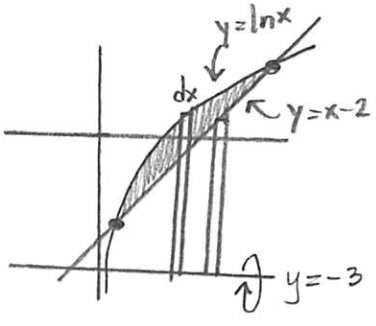
$$V=\pi\int_{y=0}^{y=1}(y^{7/5})^2\,dy$$

$$=\pi\int_0^1 y^{14/5}\,dy=\pi\left[\frac{5}{19}y^{19/5}\right]_0^1=\boxed{\frac{5\pi}{19}}$$

55.



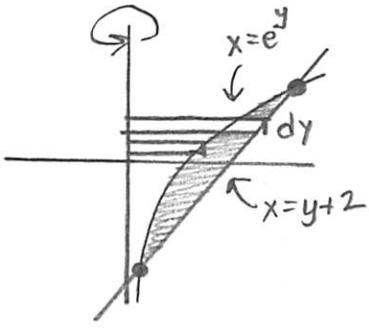
a) $$A=\int_{x=0.1586}^{x=3.1462}[\ln x-(x-2)]\,dx=\boxed{1.949}$$



b) $$V=\pi\int_{x=0.1586}^{x=3.1462}\left[(\ln x-(-3))^2-((x-2)-(-3))^2\right]dx$$

$$=\pi\int_{0.1586}^{3.1462}\left[(\ln x+3)^2-(x+1)^2\right]dx$$

$$=\boxed{34.199}$$



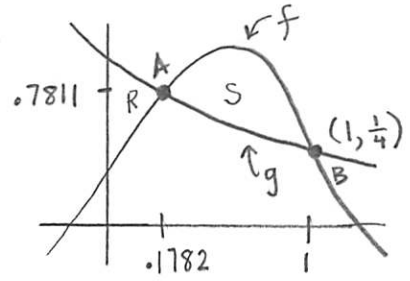
$y=\ln x$ $\quad$ $y=x-2$

$e^y=x$ $\quad$ $x=y+2$

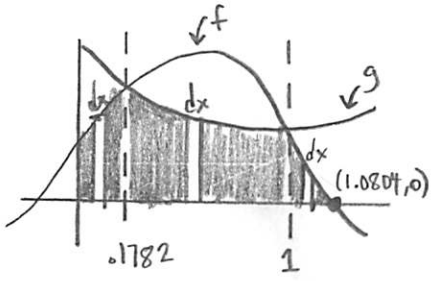
c) $$V=\pi\int_{y=-1.8414}^{y=1.1462}\left[(y+2-0)^2-(e^y-0)^2\right]dy$$

$$=\pi\int_{-1.8414}^{1.1462}\left[(y+2)^2-e^{2y}\right]dy=\boxed{5.443}$$

56.

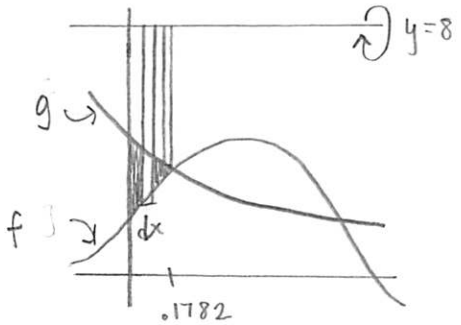


a) $A: (.1782, .7811)$
$B: (1, \frac{1}{4})$

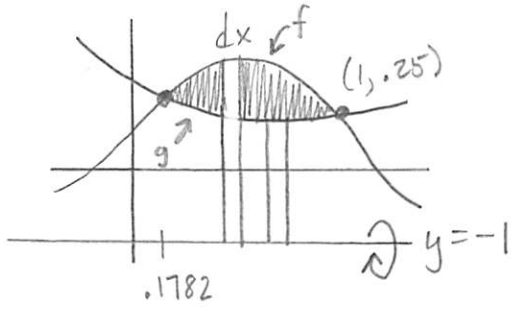


b) $$A = \int_{x=0}^{x=.1782} (f-0)\,dx + \int_{x=.1782}^{x=1} (g-0)\,dx + \int_{x=1}^{x=1.0804} (f-0)\,dx$$

$$A = .0931 + .3831 + .01 = \boxed{.4863}$$



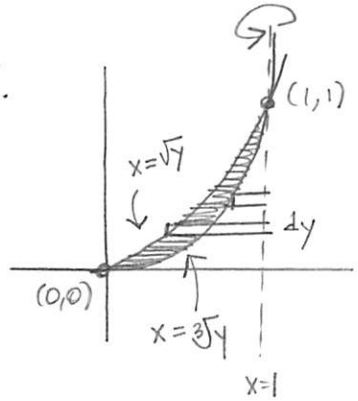
c) $$V = \pi \int_{x=0}^{x=.1782} \left[(8-f)^2 - (8-g)^2\right] dx = \boxed{2.979}$$



d) $$V = \pi \int_{x=.1782}^{x=1} \left[(f-(-1))^2 - (g-(-1))^2\right] dx$$

$$= \pi \int_{.1782}^{1} \left[(f+1)^2 - (g+1)^2\right] dx = \boxed{4.559}$$

57.



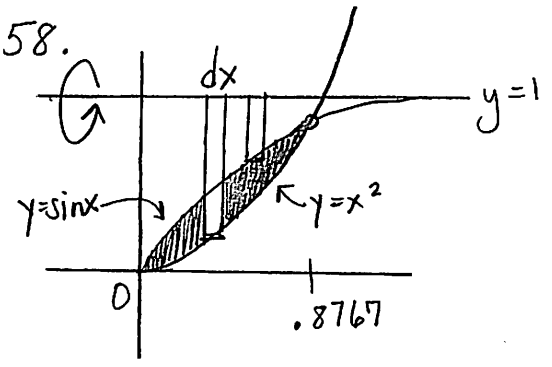
$$V = \pi \int_{y=0}^{y=1} \left[(1-\sqrt{y})^2 - (1-\sqrt[3]{y})^2\right] dy$$

$$= \pi \int_0^1 \left[1 - 2\sqrt{y} + y - (1 - 2\sqrt[3]{y} + y^{2/3})\right] dy$$

$$= \pi \left[y - \frac{4}{3}y^{\frac{3}{2}} + \frac{y^2}{2} - y + \frac{3}{2}y^{\frac{4}{3}} - \frac{3}{5}y^{\frac{5}{3}}\right]_0^1$$

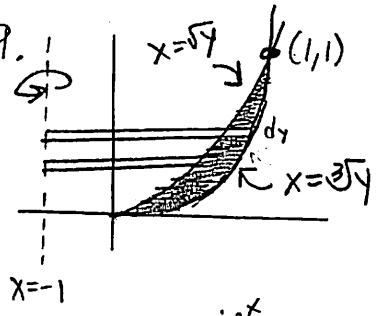
$$= \pi \left(1 - \frac{4}{3} + \frac{1}{2} - 1 + \frac{3}{2} - \frac{3}{5}\right) = \boxed{0.209}$$

58.



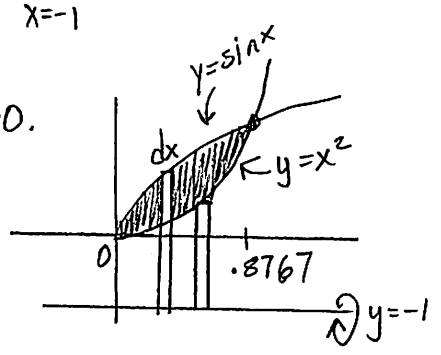
$$V=\pi\int_{x=0}^{x=.8767}\left[(1-x^2)^2-(1-\sin x)^2\right]dx = \boxed{0.573}$$

59.



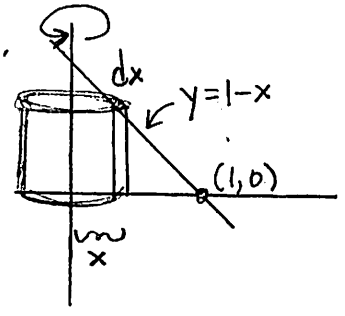
$$V=\int_{y=0}^{y=1}\left[(\sqrt[3]{y}+1)^2-(\sqrt{y}+1)^2\right]dy = \boxed{0.838}$$

60.



$$V=\pi\int_{x=0}^{x=.8767}\left[(\sin x+1)^2-(x^2+1)^2\right]dx = \boxed{1.132}$$

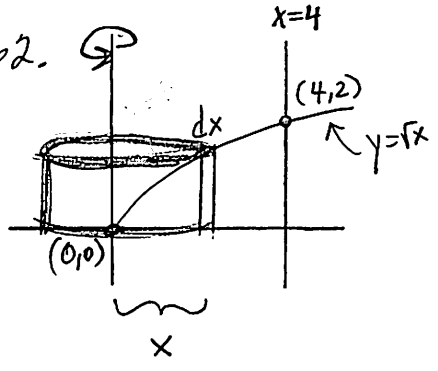
61.



$$V=2\pi\int_{x=0}^{x=1}x(1-x)\,dx = 2\pi\int_0^1(x-x^2)\,dx$$

$$=2\pi\left[\frac{x^2}{2}-\frac{x^3}{3}\right]_0^1 = 2\pi\left(\frac{1}{2}-\frac{1}{3}\right) = \boxed{\frac{\pi}{3}}$$

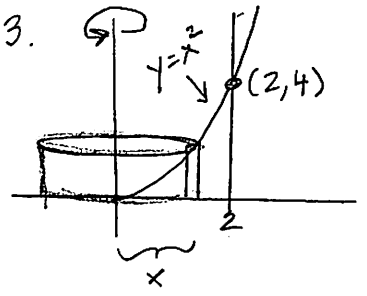
62.



$$V=2\pi\int_{x=0}^{x=4}x(\sqrt{x})\,dx = 2\pi\int_0^4 x^{\frac{3}{2}}dx$$

$$=2\pi\left[\frac{2}{5}x^{\frac{5}{2}}\right]_0^4 = 2\pi\left[\frac{2}{5}\cdot 32\right] = \boxed{\frac{128\pi}{5}}$$

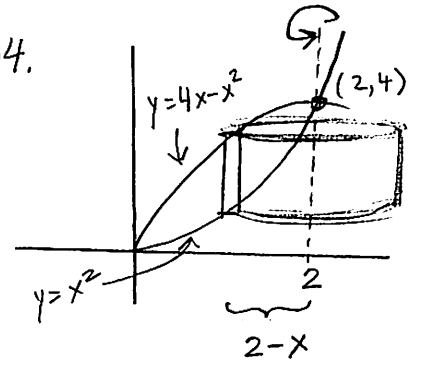
63.



$$V = 2\pi \int_{x=0}^{x=2} x(x^2)\,dx = 2\pi \int_0^2 x^3\,dx$$

$$= 2\pi \left[\frac{x^4}{4} \right]_0^2 = 2\pi(4) = \boxed{8\pi}$$

64.



$$V = 2\pi \int_{x=0}^{x=2} (2-x)(4x-x^2-x^2)\,dx$$

$$= 2\pi \int_0^2 (2-x)(4x-2x^2)\,dx$$

$$= 2\pi \int_0^2 (2-x)(2x)(2-x)\,dx$$

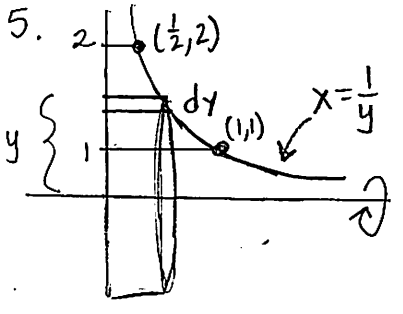
$$= 2\pi \int_0^2 2x(4-4x+x^2)\,dx$$

$$= 4\pi \int_0^2 (4x-4x^2+x^3)\,dx$$

$$= 4\pi \left[2x^2 - \frac{4}{3}x^3 + \frac{x^4}{4} \right]_0^2$$

$$= 4\pi \left(8 - \frac{32}{3} + 4 \right) = \boxed{\frac{16\pi}{3}}$$

65.



$y = \frac{1}{x}$

$x = \frac{1}{y}$

$$V = 2\pi \int_{y=1}^{y=2} y\left(\frac{1}{y}\right) dy$$

$$= 2\pi \int_1^2 dy = 2\pi [y]_1^2$$

$$= \boxed{2\pi}$$