Final Exam Review: Binomial Theorem & Vectors

Use this sheet in conjunction with your old notes, quizzes, and tests to review.

Formulas:

1. Find the term in x^4 in the expansion of $\left(3x^2 - \dfrac{2}{x}\right)^5$.

(Total 6 marks)

2. Consider the expansion of the expression $(x^3 - 3x)^6$.

(a) Write down the number of terms in this expansion.

(b) Find the term in x^{12}.

(Total 6 marks)

3. Determine the constant term in the expansion of $\left(x - \dfrac{2}{x^2}\right)^9$.

(Total 4 marks)

4. A line L passes through A(1, –1, 2) and is parallel to the line $r = \begin{pmatrix} -2 \\ 1 \\ 5 \end{pmatrix} + s \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$.

(a) Write down a vector equation for L in the form $r = a + tb$.

(2)

The line L passes through point P when $t = 2$.

(b) Find

(i) $\overrightarrow{OP}$;

(ii) $\left|\overrightarrow{OP}\right|$.

(4)
(Total 6 marks)

5. The quadrilateral $OABC$ has vertices with coordinates $O(0, 0)$, $A(5, 1)$, $B(10, 5)$ and $C(2, 7)$.

(a) Find the vectors $\overrightarrow{OB}$ and $\overrightarrow{AC}$.

(b) Find the angle between the diagonals of the quadrilateral $OABC$.

(Total 4 marks)

6. Calculate the acute angle between the lines with equations

$$r = \begin{pmatrix} 4 \\ -1 \end{pmatrix} + s\begin{pmatrix} 4 \\ 3 \end{pmatrix} \quad \text{and} \quad r = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

(Total 6 marks)

7. Let $\overrightarrow{AB} = \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix}$.

(a) Find $\overrightarrow{BC}$.

(2)

(b) Find a unit vector in the direction of $\overrightarrow{AB}$.

(3)

(c) Show that $\overrightarrow{AB}$ is perpendicular to $\overrightarrow{AB}$.

(3)

(Total 8 marks)

8. Two lines with equations $r_1 = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + s\begin{pmatrix} 5 \\ -3 \\ 2 \end{pmatrix}$ and $r_2 = \begin{pmatrix} 9 \\ 2 \\ 2 \end{pmatrix} + t\begin{pmatrix} -3 \\ 5 \\ -1 \end{pmatrix}$ intersect at the point P.

Find the coordinates of P.

(Total 6 marks)

9. The vectors $\begin{pmatrix} 2x \\ x-3 \end{pmatrix}$ and $\begin{pmatrix} x+1 \\ 5 \end{pmatrix}$ are perpendicular for two values of x.

(a) Write down the quadratic equation which the two values of x must satisfy.

(b) Find the two values of x.

(Total 4 marks)