

Binomial Distribution Practice

1. A box holds 240 eggs. The probability that an egg is brown is 0.05.

(a) Find the expected number of brown eggs in the box.

$$E(x) = 240(.05) = 12$$

(2)

(b) Find the probability that there are 15 brown eggs in the box.

$$P(X=15) = \binom{240}{15} (.05)^{15} (.95)^{225} = .073$$

(2)

(c) Find the probability that there are at least 10 brown eggs in the box.

$$P(X \geq 10) = 1 - P(X \leq 9) = .764$$

(3)

(Total 7 marks)

2. A factory makes switches. The probability that a switch is defective is 0.04. The factory tests a random sample of 100 switches.

(a) Find the mean number of defective switches in the sample.

$$\mu = 100(.04) = 4$$

(2)

(b) Find the probability that there are exactly six defective switches in the sample.

$$P(X=6) = \binom{100}{6} (.04)^6 (.96)^{94} = .105$$

(2)

(c) Find the probability that there is at least one defective switch in the sample.

$$P(X \geq 1) = 1 - P(0) = .983$$

(3)

(Total 7 marks)

3. The probability of obtaining heads on a biased coin is $\frac{1}{3}$.

(a) Sam tosses the coin three times. Find the probability of getting

(i) three heads;
$$P(X=3) = \binom{3}{3} \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^0$$
$$= .037$$

(ii) two heads and one tail.

$$P(X=2) = \binom{3}{2} \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^1 = .222 \quad (5)$$

(b) Claire plays a game in which she tosses the coin 12 times.

(i) Find the expected number of heads.

$$E(X) = 12 \left(\frac{1}{3}\right) = 4$$

(ii) Claire wins \$ 10 for each head obtained, and loses \$ 6 for each tail.

Find her expected winnings. From above, we expect 4 heads,
so 8 tails. (5)

$$(\$10 \times 4) + (-\$6 \times 8) = -\$8$$

(Total 10 marks)

4. Lily tosses a fair coin five times. Calculate the probability she obtains

(a) exactly three heads;

$$P(X=3) = \binom{5}{3} \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2 = .313$$

(b) at least one head.

$$P(X \geq 1) = 1 - P(X=0)$$
$$= 1 - \binom{5}{0} \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^5$$
$$= .969$$

(Total 6 marks)