AP Calculus AB
AP Exam Concept Study Guide

1 Functions and Trig

1. A function $y = f(x)$ is even iff $f(-x) =$ ________________.
2. Even functions are symmetric with respect to the ________________.
3. A function $y = f(x)$ is odd iff $f(-x) =$ ________________.
4. Odd functions are symmetric with respect to the ________________.
5. A function $f(x)$ is periodic with period $p > 0$ if $f(x+p) =$ ________________.
6. Functions f and g are inverses of each other iff $f(g(x)) =$________________=________________.
7. If functions f and g are inverses of each other, and $f(a) = b$, then ________________.
8. Pythagorean Identities

   (a) $\sin^2 x + \cos^2 x =$ (b) $\tan^2 x + 1 =$ (c) $\cot^2 x + 1 =$

9. Even/Odd Properties

   (a) $\sin(-\theta) =$ (b) $\cos(-\theta) =$ (c) $\tan(-\theta) =$

10. Double-Angle Formulas

    (a) $\sin 2\theta =$ (b) $\cos 2\theta =$

11. Power Reducing Formulas

    (a) $\sin^2 \theta =$ (b) $\cos^2 \theta =$

12. State the domain and range for the following functions

    (a) $f(x) = e^x$
    (d) $f(x) = \sin x$
    (g) $f(x) = \sin^{-1} x$
    (b) $f(x) = \ln x$
    (e) $f(x) = \cos x$
    (h) $f(x) = \cos^{-1} x$
    (c) $f(x) = \dfrac{1}{x}$
    (f) $f(x) = \tan x$
    (i) $f(x) = \tan^{-1} x$

2 Limits

1. A limit exists if and only if the following three criteria are met:

   (a) $\lim_{x \to a^+} f(x)$ ________________

   (b) $\lim_{x \to a^-} f(x)$ ________________

   (c) $\lim_{x \to a^+} f(x) = L =$ ________________

   In other words, the function must:

2. When evaluating a limit as $x \to a$, first try ________________________________

   (a) If substitution gives $\frac{0}{0}$, you must keep working. Try ________________________________ , or use other algebraic techniques to simplify. Then try ________________________________ ________________.

   (b) If substitution gives $\frac{0}{k}$, where k is a nonzero constant, then $\lim_{x \to a} f(x) =$ ________________.

   (c) If substitution gives $\frac{k}{0}$, where k is a nonzero constant, then $\lim_{x \to a} f(x) =$ ______, ______, or ________.

3. When evaluating a one-sided limit, first try ________________________________

   (a) If substitution gives $\frac{k}{0^+}$, where k is a positive constant, then the limit is ________________.

   (b) If substitution gives $\frac{k}{0^-}$, where k is a positive constant, then the limit is ________________.

4. When evaluating a limit as $x \to \pm\infty$, first try ________________________________

   (a) If substitution gives $\frac{0}{\pm\infty}$, then $\lim_{x \to \pm\infty} f(x) =$ ________________.

   (b) If substitution gives $\frac{k}{\pm\infty}$, where k is a constant, then $\lim_{x \to \pm\infty} f(x) =$ ________________.

   (c) If substitution gives $\frac{\pm\infty}{k}$, where k is a constant, then $\lim_{x \to \pm\infty} f(x) =$ ______, ______, or ________.

   (d) If $\lim_{x \to \pm\infty} f(x) = L$, then $y = L$ is a ________________________________ ________________________________ of f.

5. Recall the following special trig limits:

   (a) $\lim_{x \to 0} \frac{\sin x}{x} =$ _____

   (b) $\lim_{x \to 0} \frac{1 - \cos x}{x} =$ _____

3 Continuity and Differentiability

6. A function is continuous at a point c if and only if the following three criteria are met:

   (a) $f(c)$ ________________

   (b) $\lim_{x \to c} f(x)$ ________________

   (c) $\lim_{x \to c} f(x) =$ ________________

7. A function is differentiable at a point c if $\lim_{h \to 0} \dfrac{f(c+h) - f(c)}{h}$ exists. In other words, the slope must:

8. If a function is ______________________________ it must also be ______________________________ .

9. Types of discontinuity:

   (a) Removable: $\lim_{x \to a^-} f(x)$ ______ $\lim_{x \to a^+} f(x)$, but $\lim_{x \to a} f(x)$ ______ $f(a)$

   (b) Jump: $\lim_{x \to a^-} f(x)$ ______ $\lim_{x \to a^+} f(x)$

   (c) Infinite: $\lim_{x \to a^-} f(x) =$ ______ and/or $\lim_{x \to a^+} f(x) =$ ______

4 Derivatives

10. Definition of the Derivative

    - $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h} =$
    - $\lim_{x \to a} \dfrac{f(x) - f(a)}{x - a} =$

11. Basic Rules:

    - $[x^n]' =$
    - $[f \cdot g]' =$
    - $\left[\dfrac{f}{g}\right]' =$
    - $[f(g(x))]' =$

12. Derivative of an Inverse: $[f^{-1}]'(b) =$

13. Trig and Inverse Trig Derivatives

    - $[\sin u]' =$
    - $[\cos u]' =$
    - $[\tan u]' =$
    - $[\cot u]' =$
    - $[\sec u]' =$
    - $[\csc u]' =$

14. Exponential and Logarithmic Derivatives

    - $[e^u]' =$
    - $[b^u]' =$
    - $[\ln u]' =$

5 Applications of Derivatives

15. The derivative of a function is the slope of the ______________________________ line at a given point.

16. A normal line is ______________________________ to the tangent line at the point of tangency.

17. The average rate of change of $f(x)$ on the interval $[a, b] =$

18. Related Rates

    - Step 1:
    - Step 2:
    - Step 3:
    - Step 4:
    - Step 5:

19. Intermediate Value Theorem: If $f(x)$ is continuous on $[a, b]$, then for every value d between $f(a)$ and $f(b)$, there is guaranteed a value c between a and b such that: ______________________________

20. Mean Value Theorem: If $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b), then

    (a) there is guaranteed a value c between a and b such that: ______________________________

    (b) there is guaranteed a value c between a and b such that the tangent line to the curve at c is ______________________________ the secant line passing through the endpoints.

21. Rolle's Theorem: If $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b), and $f(a) = f(b)$, then

    (a) there is guaranteed a value c between a and b such that: ______________________________

    (b) there is guaranteed a value c between a and b such that the tangent line to the curve at c is ______________________________ .

22. Relationships between f, f', f''

    (a) If $f'(x) > 0$, then $f(x)$ is ______________________________

    (b) If $f'(x) < 0$, then $f(x)$ is ______________________________

    (c) If $f'(x) = 0$, then $f(x)$ could have ______________________________

    (d) If $f''(x) > 0$, then $f(x)$ is ______________________________

    (e) If $f''(x) < 0$, then $f(x)$ is ______________________________

    (f) If $f''(x) = 0$, then $f(x)$ could have ______________________________

    (g) If $f''(x) > 0$, then $f'(x)$ is ______________________________

    (h) If $f''(x) < 0$, then $f'(x)$ is ______________________________

    (i) If $f''(x) = 0$, then $f'(x)$ could have ______________________________

    (j) If $f(c)$ exists but $f'(c)$ does not exist, the graph could have a ______________________________ , ______________________________ , or _______________ at $x = c$.

23. First derivative test:

   (a) Find the critical points (where $f'(x) =$ _____ or ________)
   (b) Use the critical points to partition the domain into subintervals
   (c) Determine whether $f'(x)$ is positive or negative on each subinterval

$f'(x)$	+	-	+	+	-	-
$f(x)$						

24. Second derivative test (for concavity):

   (a) Find the critical points (where $f''(x) =$ _____ or ________)
   (b) Use the critical points to partition the domain into subintervals
   (c) Determine whether $f''(x)$ is positive or negative on each subinterval

$f''(x)$	+	-	+	+	-	-
$f'(x)$						
$f(x)$						

25. Second derivative test (for extrema):

   (a) Find the critical points (where $f'(x) =$ _____ or ________)
   (b) Use the critical points to partition the domain into subintervals
   (c) Determine whether $f''(x)$ is positive or negative at each critical point

$f''(x)$	+	-
$f'(x)$	0	0
$f(x)$		

6 Integration

26. The Fundamental Theorem of Calculus: Let $F'(x) = f(x)$, where $f(x)$ is continuous on the closed interval $[a, b]$. This means that $F(x)$ is the ______________________________ of $f(x)$, and $f(x)$ is the ______________________________ of $F(x)$.

   (a) $\int_a^b f(x)\,dx =$

   (b) $\dfrac{d}{dx} \displaystyle\int_a^{g(x)} f(t)\,dt =$

   (c) $\dfrac{d}{dx} \displaystyle\int_{h(x)}^{g(x)} f(t)\,dt =$

27. Basic Integrals

(a) $\int x^n \, dx =$ (c) $\int e^x \, dx =$ (e) $\int \ln x \, dx =$

(b) $\int \frac{1}{x} \, dx =$ (d) $\int a^x \, dx =$ (f) $\int \, dx =$

28. Trig Integrals

(a) $\int \sin u \, du =$ (c) $\int \sec^2 u \, du =$ (e) $\int \sec u \tan u \, du =$

(b) $\int \cos u \, du =$ (d) $\int \csc u \cot u \, du =$ (f) $\int \csc^2 u \, du =$

29. Properties of the Definite Integral

(a) If $\int_a^b f(x) \, dx = k$, then $\int_b^a f(x) \, dx =$ ________.

(b) If f is an even function and $\int_0^a f(x) \, dx = k$, then $\int_{-a}^a f(x) \, dx =$ ________.

(c) If f is an odd function and $\int_0^a f(x) \, dx = k$, then $\int_{-a}^a f(x) \, dx =$ ________.

7 Particle Motion

30. Position: ________________

31. Velocity: ________________ = ________________

32. Acceleration: ________________ = ________________ = ________________

33. Initial position: ________ Initial velocity: ________ Initial acceleration: ________

34. A particle is at the origin when ________________________________

35. Speed: ________________

36. Speed is increasing when

-
-

37. Speed is decreasing when

-
-

38. A particle is at rest when ________________

39. A particle is moving away from its point of origin when ________________ and is moving towards its point of origin when ________________

40. Velocity is increasing when ________________, and decreasing when ________________

41. Average velocity:
42. Average acceleration:
43. Net distance traveled:
44. Total distance traveled:
45. Position of a particle at time $t = b$:

8 Applications of Integration

46. Average Value:

47. Total Area

    (a) If $f(x) > 0$ on $[a, b]$, then $A =$

    (b) If $f(x) < 0$ on $[a, b]$, then $A =$

    (c) If $f(x) > 0$ on $[a, c]$ and $f(x) < 0$ on $[c, b]$, then $A =$

48. Area Between Two Curves

    (a) If $y = f(x)$ is above $y = g(x)$ on the interval $[a, b]$, then $A = \int_{x=a}^{b}$

    (b) If $x = f(y)$ is to the right of $x = g(y)$ on the interval $[c, d]$, then $A = \int_{y=c}^{d}$

49. Volume of Solids of Revolution

    (a) Disk Method: $V = \pi r^2 h$

    i. About the x-axis: $V = \pi \int_{x=a}^{b}$

    ii. About the y-axis: $V = \pi \int_{y=c}^{d}$

    iii. About the line $y = k$: $V = \pi \int_{x=a}^{b}$

    iv. About the line $x = k$: $V = \pi \int_{y=c}^{d}$

    (b) Washer Method: $V = \pi R^2 h - \pi r^2 h$

    i. About the x-axis: $V = \pi \int_{x=a}^{b}$

    ii. About the y-axis: $V = \pi \int_{y=c}^{d}$

    iii. About the line $y = k$: $V = \pi \int_{x=a}^{b}$

    iv. About the line $x = k$: $V = \pi \int_{y=c}^{d}$

(c) Shell Method: $V = 2\pi rh$

i. About the x-axis: $V = 2\pi \int_{y=c}^{d}$

ii. About the y-axis: $V = 2\pi \int_{x=a}^{b}$

iii. About the line $y = k$: $V = 2\pi \int_{x=a}^{b}$

iv. About the line $x = k$: $V = 2\pi \int_{y=c}^{d}$

50. Volume of Cross Sections: $V = \int_{a}^{b}$

(a) Area of square with base r:

(b) Area of square with diagonal r:

(c) Area of equilateral triangle with base r:

(d) Area of isosceles right triangle with hypotenuse r:

(e) Area of isosceles right triangle with leg r:

(f) Area of semi-circle with diameter r:

9 Calculator Use

You will need to use your calculator to do the following:

1. Graph a function within an arbitrary viewing window
2. Solve equations graphically (by finding zeros or points of intersection)
   (a) To find a zero of a function, enter the equation in Y_1 and use `[2nd] [CALC] [2:zero]`
   (b) To find a point of intersection, enter the first equation in Y_1, the second equation in Y_2, and use `[2nd] [CALC] [5:intersect]`
3. Numerically calculate the derivative of a function at a point a
   (a) `[MATH] [8]`
   (b) Old operating system: `nDeriv(function,X,a)`
4. Numerically calculate the value of a definite integral from a to b
   (a) `[MATH] [9]`
   (b) Old operating system: `fnInt(function,X,a,b)`

*Remember, you can refer to an equation stored in Y_1 by pressing

(a) `[alpha] [f4] [1:`Y_1`]`

(b) `[VARS]` → `[Y-VARS] [1:Function] [1:`Y_1`]`