KEY

AP Calculus AB
AP Exam Concept Study Guide

1 Functions and Trig

1. A function $y = f(x)$ is even iff $f(-x) =$ $f(x)$
2. Even functions are symmetric with respect to the y-axis
3. A function $y = f(x)$ is odd iff $f(-x) =$ $-f(x)$
4. Odd functions are symmetric with respect to the origin
5. A function $f(x)$ is periodic with period $p > 0$ if $f(x+p) =$ $f(x)$
6. Functions f and g are inverses of each other iff $f(g(x)) =$ $g(f(x))$ = x
7. If functions f and g are inverses of each other, and $f(a) = b$, then $f^{-1}(b) = a$
8. Pythagorean Identities

   (a) $\sin^2 x + \cos^2 x =$ 1 (b) $\tan^2 x + 1 =$ $\sec^2 x$ (c) $\cot^2 x + 1 =$ $\csc^2 x$
9. Even/Odd Properties

   (a) $\sin(-\theta) =$ $-\sin\theta$ (b) $\cos(-\theta) =$ $\cos\theta$ (c) $\tan(-\theta) =$ $-\tan\theta$
10. Double-Angle Formulas

    (a) $\sin 2\theta =$ $2\sin\theta\cos\theta$ (b) $\cos 2\theta =$ $\cos^2\theta - \sin^2\theta$
11. Power Reducing Formulas

    (a) $\sin^2\theta =$ $\frac{1-\cos 2\theta}{2}$ (b) $\cos^2\theta =$ $\frac{1+\cos 2\theta}{2}$
12. State the domain and range for the following functions

    (a) $f(x) = e^x$ D: $(-\infty, \infty)$ R: $(0, \infty)$

    (b) $f(x) = \ln x$ D: $(0, \infty)$ R: $(-\infty, \infty)$

    (c) $f(x) = \frac{1}{x}$ D: $x \neq 0$ R: $y \neq 0$

    (d) $f(x) = \sin x$ D: $(-\infty, \infty)$ R: $[-1, 1]$

    (e) $f(x) = \cos x$ D: $(-\infty, \infty)$ R: $[-1, 1]$

    (f) $f(x) = \tan x$ D: $x \neq \frac{\pi}{2} + \pi k,\ k \in \mathbb{Z}$ R: $(-\infty, \infty)$

    (g) $f(x) = \sin^{-1} x$ D: $[-1, 1]$ R: $[-\frac{\pi}{2}, \frac{\pi}{2}]$

    (h) $f(x) = \cos^{-1} x$ D: $[-1, 1]$ R: $[0, \pi]$

    (i) $f(x) = \tan^{-1} x$ D: $(-\infty, \infty)$ R: $[-\frac{\pi}{2}, \frac{\pi}{2}]$

2 Limits

1. A limit exists if and only if the following three criteria are met:

   (a) $\lim_{x\to a^+} f(x)$ exists

   (b) $\lim_{x\to a^-} f(x)$ exists

   (c) $\lim_{x\to a^+} f(x) = L =$ $\lim_{x\to a^-} f(x)$

   In other words, the function must: approach the same value L from both sides
2. When evaluating a limit as $x \to a$, first try substitution

   (a) If substitution gives $\frac{0}{0}$, you must keep working. Try factoring, or use other algebraic techniques to simplify. Then try substitution again

   (b) If substitution gives $\frac{0}{k}$, where k is a nonzero constant, then $\lim_{x\to a} f(x) =$ 0

   (c) If substitution gives $\frac{k}{0}$, where k is a nonzero constant, then $\lim_{x\to a} f(x) =$ ∞, $-\infty$, or DNE
3. When evaluating a one-sided limit, first try substitution

   (a) If substitution gives $\frac{k}{0^+}$, where k is a positive constant, then the limit is ∞

   (b) If substitution gives $\frac{k}{0^-}$, where k is a positive constant, then the limit is $-\infty$
4. When evaluating a limit as $x \to \pm\infty$, first try substitution

   (a) If substitution gives $\frac{0}{\pm\infty}$, then $\lim_{x\to\pm\infty} f(x) =$ 0

   (b) If substitution gives $\frac{k}{\pm\infty}$, where k is a constant, then $\lim_{x\to\pm\infty} f(x) =$ 0

   (c) If substitution gives $\frac{\pm\infty}{k}$, where k is a constant, then $\lim_{x\to\pm\infty} f(x) =$ ∞, $-\infty$, or DNE

   (d) If $\lim_{x\to\pm\infty} f(x) = L$, then $y = L$ is a horizontal asymptote of f.
5. Recall the following special trig limits:

   (a) $\lim_{x\to 0} \frac{\sin x}{x} =$ 1 (b) $\lim_{x\to 0} \frac{1-\cos x}{x} =$ 0

3 Continuity and Differentiability

6. A function is continuous at a point c if and only if the following three criteria are met:

   (a) $f(c)$ exists

   (b) $\lim_{x\to c} f(x)$ exists

   (c) $\lim_{x\to c} f(x) =$ $f(c)$
7. A function is differentiable at a point c if $\lim_{h\to 0} \frac{f(c+h)-f(c)}{h}$ exists. In other words, the slope must: The slope at c approaches the same value from both sides
8. If a function is differentiable it must also be continuous
9. Types of discontinuity:

   (a) Removable: $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$, but $\lim_{x\to a} f(x) \neq f(a)$

   (b) Jump: $\lim_{x\to a^-} f(x) \neq \lim_{x\to a^+} f(x)$

   (c) Infinite: $\lim_{x\to a^-} f(x) =$ $\pm\infty$ and/or $\lim_{x\to a^+} f(x) =$ $\pm\infty$

4 Derivatives

10. Definition of the Derivative

    - $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} =$ $f'(x)$
    - $\lim_{x\to a} \frac{f(x)-f(a)}{x-a} =$ $f'(c)$
11. Basic Rules:

    - $[x^n]' =$ $n x^{n-1}$
    - $[f \cdot g]' =$ $f'g + fg'$
    - $\left[\frac{f}{g}\right]' =$ $\frac{f'g - fg'}{g^2}$
    - $[f(g(x))]' =$ $f'(g) \cdot g'$
12. Derivative of an Inverse: $[f^{-1}]'(b) =$ $\frac{1}{f'(a)}$, $f(a) = b$
13. Trig and Inverse Trig Derivatives

    - $[\sin u]' =$ $\cos u \cdot u'$
    - $[\tan u]' =$ $\sec^2 u \cdot u'$
    - $[\sec u]' =$ $\sec u \tan u \cdot u'$
    - $[\cos u]' =$ $-\sin u \cdot u'$
    - $[\cot u]' =$ $-\csc^2 u \cdot u'$
    - $[\csc u]' =$ $-\csc u \cot u \cdot u'$
14. Exponential and Logarithmic Derivatives

    - $[e^u]' =$ $e^u \cdot u'$
    - $[b^u]' =$ $(b^u)(\ln b) \cdot u'$
    - $[\ln u]' =$ $\frac{1}{u} \cdot u'$

5 Applications of Derivatives

15. The derivative of a function is the slope of the tangent line at a given point.
16. A normal line is perpendicular to the tangent line at the point of tangency.
17. The average rate of change of $f(x)$ on the interval $[a, b] =$ $\frac{f(b)-f(a)}{b-a}$
18. Related Rates

    - Step 1: Draw a sketch and label all given information
    - Step 2: Write the given rate and unknown rate
    - Step 3: Write an equation relating variables from step 2. (Use secondary eq. to eliminate extra variable)
    - Step 4: Take the derivative with respect to time t
    - Step 5: Substitute all know values and solve for the unknown rate
19. Intermediate Value Theorem: If $f(x)$ is continuous on $[a, b]$, then for every value d between $f(a)$ and $f(b)$, there is guaranteed a value c between a and b such that: $f(c) = d$
20. Mean Value Theorem: If $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b), then

    (a) there is guaranteed a value c between a and b such that: $f'(c) = \frac{f(b)-f(a)}{b-a}$

    (b) there is guaranteed a value c between a and b such that the tangent line to the curve at c is parallel to the secant line passing through the endpoints.
21. Rolle's Theorem: If $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b), and $f(a) = f(b)$, then

    (a) there is guaranteed a value c between a and b such that: $f'(c) = 0$

    (b) there is guaranteed a value c between a and b such that the tangent line to the curve at c is horizontal.
22. Relationships between f, f', f''

    (a) If $f'(x) > 0$, then $f(x)$ is increasing

    (b) If $f'(x) < 0$, then $f(x)$ is decreasing

    (c) If $f'(x) = 0$, then $f(x)$ could have a max/min

    (d) If $f''(x) > 0$, then $f(x)$ is concave up

    (e) If $f''(x) < 0$, then $f(x)$ is concave down

    (f) If $f''(x) = 0$, then $f(x)$ could have a point of inflection

    (g) If $f''(x) > 0$, then $f'(x)$ is increasing

    (h) If $f''(x) < 0$, then $f'(x)$ is decreasing

    (i) If $f''(x) = 0$, then $f'(x)$ could have a max/min

    (j) If $f(c)$ exists but $f'(c)$ does not exist, the graph could have a vertical tangent, discontinuity, or cusp at $x = c$.

23. First derivative test:

(a) Find the critical points (where $f'(x) =$ 0 or DNE)

(b) Use the critical points to partition the domain into subintervals

(c) Determine whether $f'(x)$ is positive or negative on each subinterval

$f'(x)$	+	-	+	+	-	-
$f(x)$	↗	↘	↗	↗	↘	↘

max min IP max IP

24. Second derivative test (for concavity):

(a) Find the critical points (where $f''(x) =$ 0 or DNE)

(b) Use the critical points to partition the domain into subintervals

(c) Determine whether $f''(x)$ is positive or negative on each subinterval

$f''(x)$	+	-	+	+	-	-
$f'(x)$	↗	↘	↗	↗	↘	↘
$f(x)$	∪	∩	∪	∪	∩	∩

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25. Second derivative test (for extrema):

(a) Find the critical points (where $f'(x) =$ 0 or DNE)

(b) Use the critical points to partition the domain into subintervals

(c) Determine whether $f''(x)$ is positive or negative at each critical point

$f''(x)$	+	-
$f'(x)$	0	0
$f(x)$	∪	∩

min max

6 Integration

26. The Fundamental Theorem of Calculus: Let $F'(x) = f(x)$, where $f(x)$ is continuous on the closed interval $[a, b]$. This means that $F(x)$ is the antiderivative of $f(x)$, and $f(x)$ is the derivative of $F(x)$.

(a) $\int_a^b f(x)\,dx = F(b) - F(a)$

(b) $\frac{d}{dx}\int_a^{g(x)} f(t)\,dt = f(g(x)) \cdot g'(x)$

(c) $\frac{d}{dx}\int_{h(x)}^{g(x)} f(t)\,dt = f(g(x)) \cdot g'(x) - f(h(x)) \cdot h'(x)$

27. Basic Integrals

(a) $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$ (c) $\int e^x\,dx = e^x + C$ (e) $\int \ln x\,dx = x\ln x - x + C$

(b) $\int \frac{1}{x}\,dx = \ln|x| + C$ (d) $\int a^x\,dx = \frac{a^x}{\ln a} + C$ (f) $\int dx = x + C$

28. Trig Integrals

(a) $\int \sin u\,du = -\cos u + C$ (c) $\int \sec^2 u\,du = \tan u + C$ (e) $\int \sec u \tan u\,du = \sec u + C$

(b) $\int \cos u\,du = \sin u + C$ (d) $\int \csc u \cot u\,du = -\csc u + C$ (f) $\int \csc^2 u\,du = -\cot u + C$

29. Properties of the Definite Integral

(a) If $\int_a^b f(x)\,dx = k$, then $\int_b^a f(x)\,dx = -k$

(b) If f is an even function and $\int_0^a f(x)\,dx = k$, then $\int_{-a}^a f(x)\,dx = 2k$

(c) If f is an odd function and $\int_0^a f(x)\,dx = k$, then $\int_{-a}^a f(x)\,dx = 0$

7 Particle Motion

30. Position: $x(t)$

31. Velocity: $x'(t) = v(t)$

32. Acceleration: $x''(t) = v'(t) = a(t)$

33. Initial position: $p(0)$ Initial velocity: $v(0)$ Initial acceleration: $a(0)$

34. A particle is at the origin when $x(t) = 0$

35. Speed: $|v(t)|$

36. Speed is increasing when
- $v(t)$ and $a(t)$ are both positive or both negative
- graph of $v(t)$ is (1) positive and increasing or (2) negative and decreasing

37. Speed is decreasing when
- $v(t)$ and $a(t)$ have opposite signs
- graph of $v(t)$ is (1) positive and decreasing or (2) negative and increasing

38. A particle is at rest when $v(t) = 0$

39. A particle is moving away from its point of origin when $v(t) > 0$ and is moving towards its point of origin when $v(t) < 0$

40. Velocity is increasing when $a(t) > 0$, and decreasing when $a(t) < 0$.

41. Average velocity: $\frac{x(b) - x(a)}{b - a} = \frac{1}{b-a}\int v(t)\,dt$

42. Average acceleration: $\frac{v(b) - v(a)}{b - a} = \frac{1}{b-a}\int a(t)\,dt$

43. Net distance traveled: $\int_a^b v(t)\,dt$

44. Total distance traveled: $\int_a^b |v(t)|\,dt$

45. Position of a particle at time $t = b$: $x(a) + \int_a^b v(t)\,dt$

8 Applications of Integration

46. Average Value: $\frac{1}{b-a}\int_a^b f(x)\,dx$

47. Total Area

(a) If $f(x) > 0$ on $[a, b]$, then $A = \int_a^b f(x)\,dx$

(b) If $f(x) < 0$ on $[a, b]$, then $A = -\int_a^b f(x)\,dx$

(c) If $f(x) > 0$ on $[a, c]$ and $f(x) < 0$ on $[c, b]$, then $A = \int_a^c f(x)\,dx - \int_c^b f(x)\,dx$

48. Area Between Two Curves

(a) If $y = f(x)$ is above $y = g(x)$ on the interval $[a, b]$, then $A = \int_{x=a}^b [f(x) - g(x)]\,dx$

(b) If $x = f(y)$ is to the right of $x = g(y)$ on the interval $[c, d]$, then $A = \int_{y=c}^d [f(y) - g(y)]\,dy$

49. Volume of Solids of Revolution

(a) Disk Method: $V = \pi r^2 h$

i. About the x-axis: $V = \pi\int_{x=a}^b r^2\,dx$, $r = f(x)$

ii. About the y-axis: $V = \pi\int_{y=c}^d r^2\,dy$, $r = f(y)$

iii. About the line $y = k$: $V = \pi\int_{x=a}^b (r - k)^2\,dx$, $r = f(x)$

iv. About the line $x = k$: $V = \pi\int_{y=c}^d (r - k)^2\,dy$, $r = f(y)$

(b) Washer Method: $V = \pi R^2 h - \pi r^2 h$

i. About the x-axis: $V = \pi\int_{x=a}^b (R^2 - r^2)\,dx$

ii. About the y-axis: $V = \pi\int_{y=c}^d (R^2 - r^2)\,dy$

iii. About the line $y = k$: $V = \pi\int_{x=a}^b [(f_1(x) - k)^2 - (f_2(x) - k)^2]\,dx$

iv. About the line $x = k$: $V = \pi\int_{y=c}^d [(f_1(y) - k)^2 - (f_2(y) - k)^2]\,dy$

(c) Shell Method: $V = 2\pi r h$

i. About the x-axis: $V = 2\pi\int_{y=c}^d y\,(\text{right} - \text{left})\,dy$

ii. About the y-axis: $V = 2\pi\int_{x=a}^b x\,(\text{top} - \text{bottom})\,dx$

iii. About the line $y = k$: $V = \pi\int_{x=a}^b (k - f(x))(\text{right} - \text{left})\,dy$

iv. About the line $x = k$: $V = \pi\int_{y=c}^d (k - f(y))(\text{top} - \text{bottom})\,dx$

50. Volume of Cross Sections: $V = \int_a^b A(x)\,dx$

(a) Area of square with base r: $A = r^2$

(b) Area of square with diagonal r: $A = \frac{1}{2}r^2$

(c) Area of equilateral triangle with base r: $A = \frac{\sqrt{3}}{4}r^2$

(d) Area of isosceles right triangle with hypotenuse r: $A = \frac{1}{4}r^2$

(e) Area of isosceles right triangle with leg r: $A = \frac{1}{2}r^2$

(f) Area of semi-circle with diameter r: $A = \frac{\pi}{8}r^2$

9 Calculator Use

You will need to use your calculator to do the following:

1. Graph a function within an arbitrary viewing window

2. Solve equations graphically (by finding zeros or points of intersection)

(a) To find a zero of a function, enter the equation in Y_1 and use `[2nd] [CALC] [2:zero]`

(b) To find a point of intersection, enter the first equation in Y_1, the second equation in Y_2, and use `[2nd] [CALC] [5:intersect]`

3. Numerically calculate the derivative of a function at a point a

(a) `[MATH] [8]`

(b) Old operating system: `nDeriv(function,X,a)`

4. Numerically calculate the value of a definite integral from a to b

(a) `[MATH] [9]`

(b) Old operating system: `fnInt(function,X,a,b)`

*Remember, you can refer to an equation stored in Y_1 by pressing `[VARS]` → `[Y-VARS]` `[1:Function]` `[1:Y1]`