AP Calculus AB
Unit 7 Review

Name: ______________________

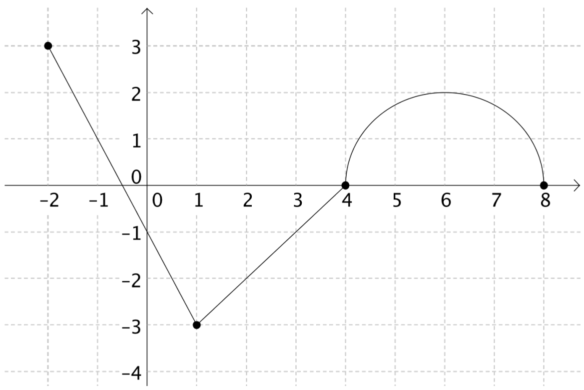
Block: __________ Date: __________

1. $\dfrac{d}{dx}\int_{1}^{x} \sqrt[5]{t^2-1}\,dt$

2. $\dfrac{d}{dx}\int_{x^2}^{0} (3t-1)\,dt$

3. $\dfrac{d}{dx}\int_{\pi}^{\sin x} \cos\sqrt{t}\,dt$

4. Given below is the graph of $f(t)$ and the function $g(x)$ is defined to be $g(x) = \int_{-2}^{2x} f(t)\,dt$



(a) Find the value of $g(-1)$.

(b) Find the value of $g'(-1)$.

(c) Find the value of $g(2)$.

(d) Find the value of $g'(2)$.

(e) Find the value of $g''(2)$.

(f) Find the value of $g'(-1/2)$.

(g) Find the value of $g''(-1/2)$.

5. $\displaystyle\int \frac{\tan(4x)}{\cos^2(4x)}\,dx$

6. $\displaystyle\int \frac{6x^2\sec^2(x^3)}{\tan(x^3)}\,dx$

7. $\displaystyle\int 4x\sqrt[3]{3x^2-12}\,dx$

8. $\displaystyle\int \frac{x+3}{\sqrt{2x+1}}\,dx$

9. $\displaystyle\int 8e^{1-4x}\,dx$

10. $\displaystyle\int \frac{3\ln(x+1)}{2x+2}\,dx$

11. Let $P(t)$ represent the number of wolves in a population at time t years, when $t \geq 0$. The population $P(t)$ is increasing at a rate directly proportional to $800 - P(t)$, where the constant of proportionality is k.

(a) If $P(0) = 500$, find $P(t)$ in terms of t and k.

(b) If $P(2) = 700$, find k.

(c) Find $\lim_{t\to\infty} P(t)$.

12. Solve the differential equation $\dfrac{dy}{dx} = y^2(6-2x)$.

13. Find the general solution to the differential equation $e^{-y}\sin x - y'\cos^2 x = 0$.

14. Find the particular solution to the differential equation $\dfrac{dy}{dx} = \dfrac{3x^2}{e^{2y}}$ with the initial condition $f(0) = 2$.

15. Find the particular solution to $\dfrac{dy}{dx} = 1 - y + x^2 - yx^2$ with the initial condition $f(0) = -4$.

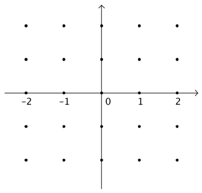
16. Verify that $x = 3t^2 + 1$ is a solution to the differential equation $2x - x't + 4 = x''$.

17. Consider the slope field for the differential equation $\dfrac{dy}{dx} = \dfrac{e^{2y-1}}{x+1}$.

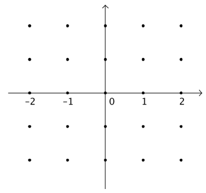
1. Describe all points for which the slope field has horizontal segments.
2. Describe all points for which the slope field has vertical segments.
3. Describe all points for which $\dfrac{dy}{dx} = 1$.

Sketch a slope field for the given differential equations at the indicated points

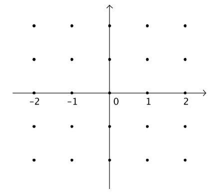
18. $y' = 0.5x - 1$



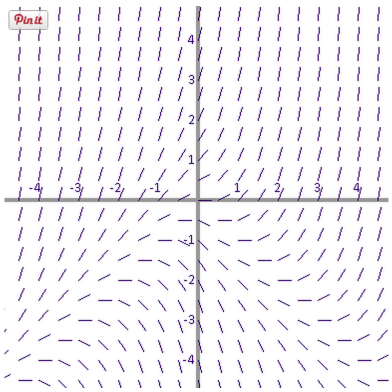
19. $y' = 0.5y$



20. $y' = -\dfrac{x}{y}$

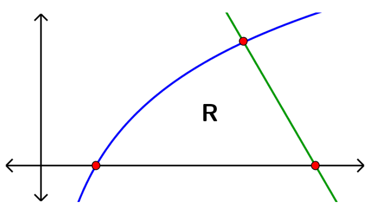


Sketch the particular solution to the differential equation represented by the slope field below.



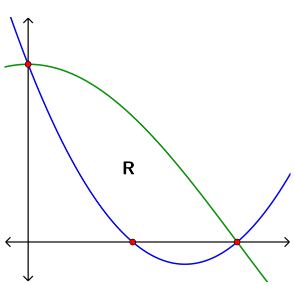
21. $f(3) = 0$
22. $f(0) = -2$
23. $f(-2) = 0$

24. (CALC) Let R be the region in the first quadrant bounded by the x-axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown below.



   (a) Write and evaluate an integral to find the area of R.
   (b) Region R is the base of a solid. For the solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.
   (c) The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.

25. Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos(\frac{1}{4}\pi x)$. Let R be the region bounded by the graphs of f and g, as shown in the figure below.



   (a) Without using your calculator, write and evaluate an integral expression to find the area of R.
   (b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 4$.
   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.