AP Calculus AB / IB SL II - Unit 6

1. $\int(\sqrt[3]{x}+x)\,dx = \int(x^{\frac{1}{3}}+x)\,dx = \boxed{\frac{3}{4}x^{\frac{4}{3}}+\frac{1}{2}x^2+C}$

2. $\int(2x-3x^2)\,dx = \boxed{x^2-x^3+C}$

3. $\int x^2(2x^2+3x)\,dx = \int(2x^4+3x^3)\,dx = \boxed{\frac{2}{5}x^5+\frac{3}{4}x^4+C}$

4. $\int(x^{3/2}+2x+1)\,dx = \boxed{\frac{2}{5}x^{5/2}+x^2+x+C}$

5. $\int\left(\sqrt{x}+\frac{1}{2\sqrt{x}}\right)dx = \int\left(x^{\frac{1}{2}}+\frac{1}{2\sqrt{x}}\right)dx = \boxed{\frac{2}{3}x^{\frac{3}{2}}+\sqrt{x}+C}$

6. $\int\frac{3x^2+x+3}{x^3}\,dx = \int\left(\frac{3}{x}+\frac{1}{x^2}+\frac{3}{x^3}\right)dx = \int(3x^{-1}+x^{-2}+3x^{-3})\,dx$

$= 3\ln|x| - x^{-1} - \frac{3}{2}x^{-2}+C = \boxed{3\ln|x|-\frac{1}{x}-\frac{3}{2x^2}+C}$

7. $\int y^3\sqrt{y}\,dy = \int y^{7/2}\,dy = \boxed{\frac{2}{9}y^{9/2}+C}$

8. $\int\frac{1}{w\sqrt{w}}\,dw = \int w^{-3/2}\,dw = -w^{-1/2}+C = \boxed{-\frac{1}{\sqrt{w}}+C}$

9. $\int\frac{x^3+3}{\sqrt{x}}\,dx = \int\left(x^{5/2}+3x^{-\frac{1}{2}}\right)dx = \boxed{\frac{2}{7}x^{7/2}+6\sqrt{x}+C}$

10. $\int(x+3)(x-3)^2\,dx = \int(x+3)(x^2-6x+9)\,dx = \int(x^3-6x^2+9x+3x^2-18x+27)\,dx$

$= \int(x^3-3x^2-9x+27)\,dx = \boxed{\frac{x^4}{4}-x^3-\frac{9}{2}x^2+27x+C}$

11. $\int(\theta^2+\cos\theta)\,d\theta = \boxed{\frac{1}{3}\theta^3+\sin\theta+C}$

12. $\int(\sqrt{x}-\sin x+2)\,dx = \boxed{\frac{2}{3}x^{\frac{3}{2}}+\cos x+2x+C}$

13 $f'(x) = 2x - \sin x$

$f(x) = \int (2x - \sin x)\,dx = x^2 + \cos x + C$

$f(0) = 4 = 0 + 1 + C \Rightarrow C = 3$

$f(x) = \boxed{x^2 + \cos x + 3}$

14 $f''(x) = x^2$

$f'(x) = \int x^2\,dx = \frac{1}{3}x^3 + C$

$f'(0) = 6 = C$

$f'(x) = \frac{1}{3}x^3 + 6$

$f(x) = \int \left(\frac{1}{3}x^3 + 6\right)dx = \frac{1}{12}x^4 + 6x + C$

$f(0) = 3 = C$

$f(x) = \boxed{\frac{1}{12}x^4 + 6x + 3}$

15 $\int_{-3}^{1} h(x)\,dx \approx 5(2) + 2(2) = \boxed{14}$

16 $\int_{-3}^{9} h(x)\,dx \approx -3(4) + (-2)(4) + 11(4) = \boxed{24}$

17 $\int_{-3}^{9} h(x)\,dx \approx 2(4) - 7(4) + 6(4) = \boxed{4}$

18 $\int_{-3}^{3} h(x)\,dx \approx \frac{5+2}{2}(2) + \frac{2-3}{2}(2) + \frac{-3-7}{2}(2) = \boxed{-4}$

19 $\int_{-3}^{9} h(x)\,dx \approx \frac{1}{2}(2)\big(5 + 2(2) + 2(-3) + 2(-7) + 2(-2) + 2(6) + 11\big)$

$= (5 + 4 - 6 - 14 - 4 + 12 + 11) = \boxed{8}$

20

0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
0	$\frac{\pi}{2\sqrt{2}}$	π	$\frac{3\pi}{2\sqrt{2}}$	0

$$\int_0^{\pi} 2x\sin x\,dx \approx \frac{\pi}{4}\left(\frac{\pi}{2\sqrt{2}}\right) + \frac{\pi}{4}(\pi) + \frac{\pi}{4}\left(\frac{3\pi}{2\sqrt{2}}\right) + \cancel{\frac{\pi}{4}(0)} = \boxed{5.957}$$

21

-1	1.75	4.5	7.25	10
7.389	22.629	$149.6\cancel{38}$	388.387	738.906

$$\frac{1}{2}(2.75)\left[7.389 + 2(22.629) + 2(149.628) + 2(388.387) + 738.906\right]$$

$$= 2567.939$$

22 $8(3) + (-4)(4) + 2(4) = 24 - 16 + 8 = \boxed{16}$

23 $\left(\frac{5+8}{2}\right)(2) + \left(\frac{8+2}{2}\right)(1) + \left(\frac{2-4}{2}\right)(2) + \left(\frac{-4-1}{2}\right)(2) + \left(\frac{-1+2}{2}\right)(3) + \left(\frac{2+5}{2}\right)(1)$

$$= 13 + 10 - 2 - 5 + \frac{3}{2} + \frac{7}{2} = \boxed{21}$$

24 $\int_{-1}^{1} (t^2 - t)\,dt = \left[\frac{t^3}{3} - \frac{t^2}{2}\right]_{-1}^{1} = \left(\frac{1}{3} - \frac{1}{2}\right) - \left(-\frac{1}{3} - \frac{1}{2}\right) = \boxed{\frac{2}{3}}$

25 $\int_1^2 \left(\frac{3}{x^2} - 1\right)dx = \int_1^2 (3x^{-2} - 1)\,dx = \left[-3x^{-1} - x\right]_1^2$

$$= \left[-\frac{3}{x} - x\right]_1^2 = \left(-\frac{3}{2} - 2\right) - (-3 - 1) = \boxed{\frac{1}{2}}$$

26 $\int_1^4 \frac{u-2}{\sqrt{u}}\,du = \int_1^4 \left(\sqrt{u} - \frac{2}{\sqrt{u}}\right)du = \left[\frac{2}{3}u^{\frac{3}{2}} - 4\sqrt{u}\right]_1^4$

$$= \left(\frac{2}{3}(4^{3/2}) - 4\sqrt{4}\right) - \left(\frac{2}{3} - 4\right) = \frac{16}{3} - 8 - \frac{2}{3} + 4 = \boxed{\frac{2}{3}}$$

27 $\int_{-1}^{-2} \left(x - \frac{1}{x^2}\right)dx = \left[\frac{x^2}{2} + \frac{1}{x}\right]_{-1}^{-2} = \left(\frac{4}{2} + \frac{1}{-2}\right) - \left(\frac{1}{2} - 1\right) = \boxed{2}$

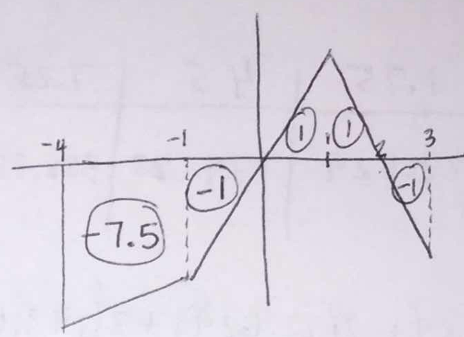
28 $\int_0^{\pi}(1+\sin x)\,dx = \left[x-\cos x\right]_0^{\pi} = (\pi+1)-(0-1) = \boxed{\pi+2}$

29 $\int_1^3(3x^2+5x-4)\,dx = \left[x^3+\frac{5}{2}x^2-4x\right]_1^3 = \left(27+\frac{45}{2}-12\right)-\left(1+\frac{5}{2}-4\right)$

$= \boxed{38}$

30 $\int_{-4}^{2} f(x)\,dx = \boxed{-6.5}$

31 $\int_0^3 f(x)\,dx = \boxed{1}$



32 $\int_{-1}^{1} f(x)\,dx = \boxed{0}$

33 $\int_{-4}^{0} f'(x)\,dx = f(0)-f(-4) = \boxed{3}$

34 $\int_{-1}^{1} f'(x)\,dx = f(1)-f(-1) = 2-(-2) = \boxed{4}$

35 $\int_1^3 f'(x)\,dx = f(3)-f(1) = -2-2 = \boxed{-4}$

36 a) $\int_2^6 f(x)\,dx + \int_2^6 g(x)\,dx = 10-2 = \boxed{8}$

b) $2\int_2^6 f(x)\,dx - 3\int_2^6 g(x)\,dx = 2(10)-3(-2) = \boxed{26}$

c) $6\int_6^2 g(x)\,dx = -6\int_2^6 g(x)\,dx = -6(-2) = \boxed{12}$

d) $\int_2^6 \frac{g(x)}{2f(x)}\,dx$ cannot be determined

37 a) $\int_{-2}^{4} f(x)\,dx + \int_{-2}^{4} 4\,dx = -6+\left[4x\right]_{-2}^{4} = -6+(16+8) = \boxed{18}$

b) $3\int_{-2}^{4} g(x)\,dx + \int_{-2}^{4} x\,dx = 3(4)+\left[\frac{x^2}{2}\right]_{-2}^{4} = 12+(8-2) = \boxed{18}$

c) $\frac{1}{2}\int_{-2}^{4} f(x)\,dx + 3\int_{-2}^{4} x^2\,dx = -3+\cancel{3}\left[\frac{x^3}{\cancel{3}}\right]_{-2}^{4} = -3+64+8 = \boxed{69}$

38 a) $\int_0^7 f'(x)dx = f(7)-f(0) = 3+1.5+2+2.5 = \boxed{9}$

b) $\int_0^3 f'(x)dx = f(3)-f(0) = f(3)+3 = 3 \quad \therefore f(3)=\boxed{0}$

c) $\int_3^7 f'(x)dx = f(7)-f(3) = f(7)+1 = 6 \quad \therefore f(7)=\boxed{5}$

39 a) $\int_{-2}^0 f'(x)dx = f(0)-f(-2) = f(0)-5 = 4 \quad \therefore f(0)=\boxed{9}$

b) $\int_{-2}^6 f'(x)dx = f(6)-f(-2) = f(6)-5 = 2\pi \quad \therefore f(6)=\boxed{2\pi+5}$

40 a) moving left on $(\frac{25}{3}, 10]$ since $v(t)<0$

moving right on $[0, \frac{25}{3})$ since $v(t)>0$

b) total distance $= \int_0^7 |v(t)|dt = 9+16-2\pi = \boxed{25-2\pi}$ m

c) $\int_5^7 v(t)dt + \int_7^{10} v(t)dt = 8-\pi+\int_7^{10}(-3x+25)dx$

$= 8-\pi+\left[-\frac{3x^2}{2}+25x\right]_7^{10} = 8-\pi-\frac{3}{2} = 6.5-\pi = \boxed{3.358}$ m

d) $a(2) = v'(2) = \boxed{\frac{2}{3}\ m/s^2}$

e) $\int_0^5 v(t)dt + p(0) = 9+8-\pi+12 = \boxed{29-\pi}$ m

41 a) $[0,1)\cup(1,3)$ since $v>0$

b) $(3,4]$ since $v<0$

c) $(1,2)\cup(3,4)$ since $|v|$ is increasing

d) $(0,1)\cup(2,3)$ since $|v|$ is decreasing

e) $p(4)-p(0) = \int_0^4 v(t)dt = \boxed{2-\frac{\pi}{2}}$

f) $\int_0^3 v(t)dt - \int_3^4 v(t)dt = 2-\frac{\pi}{2}+\frac{1}{2}-(-\frac{1}{2}) = \boxed{3-\frac{\pi}{2}}$

42 a) The total distance the car travels from $t=0$ to $t=60$ sec.

$30(25) + 14(10) + \cancel{0(15)} = \boxed{890\text{ ft}}$

b) $\int_{15}^{50} a(t)\,dt = v(50) - v(15)$ is the change in velocity from $t=15$ to $t=50$ sec.

c) $\boxed{\text{Yes}}$ if $a(t)$ is continuous on $[0,30]$ and differentiable on $(0,30)$ then Rolle's Thm guarantees this since $a(0)=a(30)$

d) $v'(31) \approx \dfrac{a(35)-a(30)}{35-30} = \dfrac{2-1}{5} = \boxed{\dfrac{1}{5}\text{ ft/sec}^2}$

The velocity is increasing at $t=31$ sec.

e) $\frac{1}{35}\int_{25}^{60} a(t)\,dt$ gives the average acceleration from $t=25$ sec to $t=60$ sec

43 a) $\int_0^4 S(t)\,dt = \boxed{21.173}$ is the amount of sand added from $t=0$ to $t=4$ hrs, in pounds.

b) $\int_0^4 R(t)\,dt = \boxed{19.505}$ pounds is the amount of sand used from $t=0$ to $t=4$ hrs.

c) $\frac{1}{4}\int_0^4 S(t)\,dt = \boxed{5.293\text{ pounds/hr}}$ is the average rate that sand is being added from $t=0$ to $t=4$ hrs.

d) $A(t) = 120 + \int_0^t [S(t) - R(t)]\,dt$

e) $A(7) = 120 + \int_0^7 [S(t) - R(t)]\,dt = \boxed{114.176}$ pounds

f) $114.176 + \int_7^K S(t)\,dt = 200$

$\int_7^K S(t)\,dt = 85.823$

44 a) $\int_0^6 R(t)\,dt = \boxed{31.816 \text{ yd}^3}$

b) $Y(t) = 2500 + \int_0^t [S(t) - R(t)]\,dt$

45 $12(8) + 13(9) + 17(5) + 22(8) = \boxed{474}$

46 $V(0) + \int_0^{4.628} a(t)\,dt$

47 $62(6) + 60(6) = \boxed{732}$ gallons

48 $\int_a^b f(x)\,dx + \int_a^b 3\,dx = 2a - 3b + \left[3x\right]_a^b$

$= 2a - 3b + 3b - 3a = \boxed{-a}$

49 a) $\int_1^3 f(x)\,dx = F(3) - F(1) = 8 - 3 = \boxed{5}$

b) $\int_0^2 f(x)\,dx = \int_2^3 f(x)\,dx$

$F(2) - F(0) = F(3) - F(2)$

$A - 4 = 8 - A$

$2A = 12$

$\boxed{A = 6}$