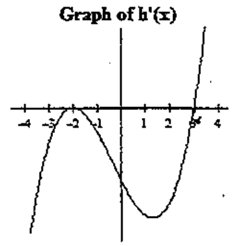
5. Pictured to th[...]t is the graph of $h'(x)$, the derivative of a function, [...](x). Which of the following statements is/are true? $(h'(x)=0)$

   I. The graph of $h(x)$ has a horizontal tangent when $x = 3$. ✓

   II. The graph of $h(x)$ is increasing on the interval $(-\infty, 2)$. X

   III. The graph of $h(x)$ is decreasing on the interval $(-\infty, 3)$. ✓X → yes, except at $x=-2$



A. I only
B. II only
C. I and II only
D. I and III only
E. I, II, and III

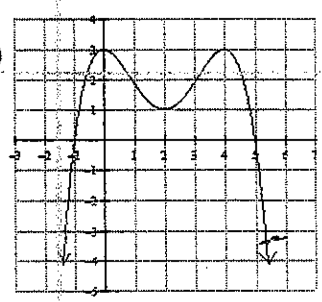
6. Find $\lim_{h\to 0} \frac{\sin(x+h)-\sin x}{h} = f'(x)$ $\left\{ \begin{array}{l} f(x) = \sin x \\ f'(x) = \cos x \end{array} \right.$

A. $\cos x$
B. $\sin x$
C. $-\sin x$
D. $-\cos x$
E. Limit does not exist

7. The graph of a quartic function, $p(x)$, is pictured to the right
On which of the following intervals is $p'(x) < 0$? (negative slope)

   I. $(0, 2)$ ✓ II. $(-\infty, 0)$ X

   III. $(4, \infty)$ ✓ IV. $(2, 4)$ X



A. II and IV only
B. I and III only
C. III only
D. IV only
E. I only

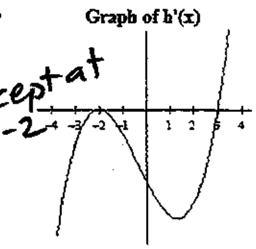
Key

5. Pictured to the right is the graph of $h'(x)$, the derivative of a function, $h(x)$. Which of the following statements is/are true?

   I. The graph of $h(x)$ has a horizontal tangent when $x = 3$. ✓

   II. The graph of $h(x)$ is decreasing on the interval $(-\infty, 3)$. ✓X → except at $x=-2$

   III. The graph of $h(x)$ is increasing when $x = 4$. ✓



A. I only
B. II only
C. I and II only
D. I, II, and III
E. I and III only

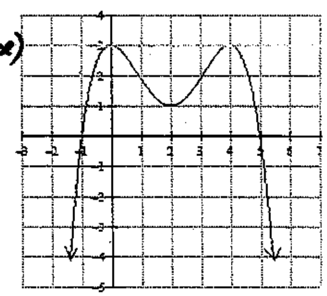
6. Find $\lim_{h\to 0} \frac{\cos(x+h)-\cos x}{h} = f'(x)$ $\quad f(x) = \cos x$, $f'(x) = -\sin x$

A. $\cos x$
B. $\sin x$
C. $-\sin x$
D. $-\cos x$
E. Limit does not exist

7. The graph of a quartic function, $p(x)$, is pictured to the right
On which of the following intervals is $p'(x) > 0$? (positive slope)

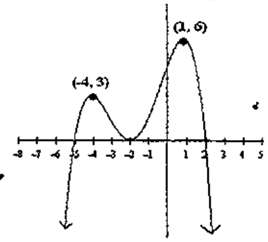
   I. $(0, 2)$ X II. $(-\infty, 0)$ ✓

   III. $(4, \infty)$ X IV. $(2, 4)$ ✓



A. I and III only
B. II and III only
C. III only
D. II and IV only
E. I only

6. The graph pictured to the right is the graph of $f'(x)$, the derivative of a polynomial function, $f(x)$. Which of the following statements is/are true?



I. $f(x)$ is increasing on the interval $(-5, 2)$. ✓

II. $f(x)$ has a relative minimum when $x = -5$. ✓

III. The slope of the normal line drawn to $f(x)$ at $x = -4$ is $-\frac{1}{3}$. ✓

A. II only B. I and II only C. III only

D. I, II, and III E. II and III only

7. If $g'(x) = -3x(x+2)^2$, then the graph of $g(x)$ has a relative maximum at what value(s) of x?

A. 0 only

B. -2 and 0 only

C. -2 only

D. -2 and $-\frac{2}{3}$ only

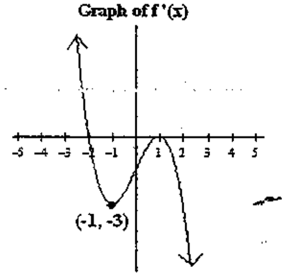
E. $g(x)$ never reaches a relative maximum

+ + −
−2 0

Free Response

Pictured to the right is the graph of the first derivative, $f'(x)$, of a polynomial function $f(x)$, such that $f(-1) = 2$.

Graph of f '(x)



a. Approximate the value of $f(-0.9)$ using the equation of the tangent line drawn to the graph of $f(x)$ when $x = -1$.

b. On what interval(s) is the graph of $f(x)$ increasing or decreasing? Give a reason for your answer.

c. At what value(s) of x does the graph of $f(x)$ have a relative maximum? Justify your answer.

d. At what value(s) of x does the graph of $f(x)$ have a relative minimum? Justify your answer.

a. $y - 2 = -3(x+1)$
$y - 2 = -3(-.9+1)$
$y = 1.7$

b. inc: $(-\infty, -2)$
dec: $(-2, 1) \cup (1, \infty)$

c. f' + − −
−2 1
rel max at $x = -2$ since f' changes from + to −.

d. No rel min since f' never changes from − to +.

AP Calculus
Test #2: Unit #2 – Understanding the Derivative

A GRAPHING CALCULATOR IS NOT ALLOWED FOR THIS SECTION OF THE EXAM.

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Multiple Choice

The derivative, g', of a polynomial function g is continuous and has exactly two zeros. Selected values of g' are given in the table below. Use the table to answer questions 1 – 3.

x	−4	−3	−2	−1	0	1	2	3	4
$g'(x)$	2	3	0	−3	−2	−1	0	3	2

1. On which of the following intervals is the graph of $g(x)$ increasing? ($g'(x)$ is positive)

I. $(-\infty, -2)$ ✓ II. $(-2, 2)$ ✗ III. $(2, \infty)$ ✓

A. I only B. II only C. III only D. I and III only E. I and II only

2. At what value(s) of x does the graph of $g(x)$ reach a relative minimum?

g' + − +
−2 2

A. -2 only B. -2 and 2 only C. 2 only
D. 0 and 2 only E. 0 only

3. Which of the following is the tangent line approximation of $g(2.9)$ if $g(3) = -2$?

A. -2.3
B. 1.7
C. 3.1
D. -2.1
E. -1.7

$(3, -2)$
$g'(3) = 3$ from table, so
$y + 2 = 3(x - 3)$
$y + 2 = 3(2.9 - 3)$
$y = -2.3$

14. A rodeo pe[illegible]er spins a lasso in a circle perpendicular to the ground. The height from the ground of the knot, measured in units of feet, in the lasso is modeled by the function

$$H(t) = -3\cos\left(\frac{5\pi}{3}t\right) + 5,$$

where t is the time measured in seconds after the lasso begins to spin.

a. Find the value of $H(0.75)$. Using correct units, explain what this value represents in the context of this problem.

H(.75) = 7.121 feet. This is the height of the lasso at .75 seconds.

b. Find the value of $H'(0.75)$. Using correct units, explain what this value represents in the context of this problem.

H'(.75) = -11.11 ft/sec. This is the rate at which the height of the lasso is changing.

AP Calculus AB
Quiz #5
Graphing Calculator Permitted

MULTIPLE CHOICE

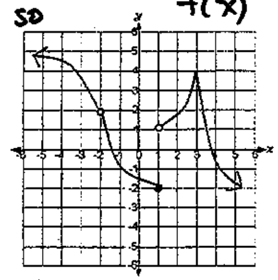
1. Which of the following statements can be made about the graph of the function $h(x) = \frac{\ln(\cos x)}{\tan x}$ when $x = \frac{\pi}{2}$.

A. The graph of $h(x)$ is increasing.
B. The graph of $h(x)$ is decreasing.
C. The graph of $h(x)$ has a vertical tangent.
D. The graph of $h(x)$ has a horizontal tangent.
E. No conclusion can be made about the graph of $h(x)$.

2. Consider the graph of $f(x)$ to the right to determine which of the following statements is/are true.

I. $f'(x) = 0$ when $x = 3$. → nope, its a cusp so f' dne
II. $f'(2.5) > 0$. ✓ positive slope
III. On the interval $(-4,5)$ there are three values of x at which $f(x)$ is not differentiable. ✓ → not diff. at $x = -2, 1, \& 3$

f(x)



A. I only
B. I and II only
C. III only
D. II and III only
E. I, II and III

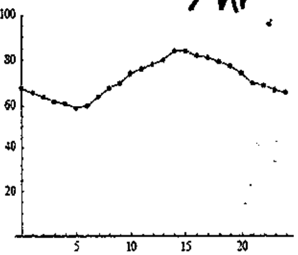
3. Let $f(7) = 0$, $f'(7) = 14$, $g(7) = 1$ and $g'(7) = \frac{1}{7}$. Find $h'(7)$ if $h(x) = \frac{f(x)}{g(x)}$.

$$h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

$$h'(7) = \frac{(1)(14) - (0)(\frac{1}{7})}{1^2} = 14$$

A. 98
B. −14
C. −2
D. 14
E. Cannot be determined

4. The graph to the right shows data of a function, $H(t)$, which shows the relationship between temperature in °C (y-axis) and the time in hours (x-axis). What does the value of $H'(6)$ represent? ↳ °C/hr.



~~A.~~ $H'(6)$ represents the temperature after 6 hours measured in °C.

~~B.~~ $H'(6)$ represents the rate at which the temperature is changing after 6 hours measured in °C.

~~C.~~ $H'(6)$ represents the temperature after 6 hours measured in °C per hour.

D. $H'(6)$ represents the rate at which the temperature is changing after 6 hours measured in °C per hour.

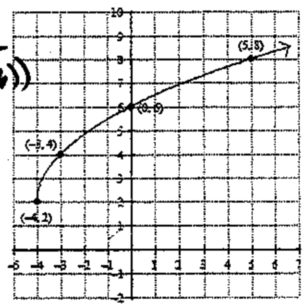
~~E.~~ $H'(6)$ represents the amount of temperature change over the first 6 hours measured in °C

5. The graph of $h(x) = 2\sqrt{x+4}+2$ is pictured below. What is the value of $[h^{-1}(6)]'$?

Let $g(x)$ be inverse of h.

$[h^{-1}(6)]' = g'(6) = \frac{1}{h'(g(6))} = \frac{1}{h'(0)} = \frac{1}{\frac{1}{2}} = 2$

$g(6) = \boxed{0}$
$h(\boxed{0}) = 6$



A. $\frac{1}{4}$
B. 2
C. 4
D. $\frac{1}{2}$
E. 0

6. Find y' if $y = x^2e^x$.

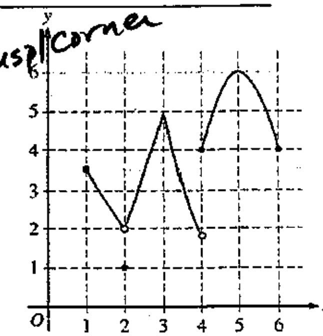
$y = x^2 \cdot e^x + e^x \cdot 2x$

A. $2xe^x$
B. $x(x+2e^x)$
C. $xe^x(x+2)$
D. $2x+e^x$
E. $2x+e$

7. The function f is pictured to the right. At which of the following values of x is f defined and continuous but not differentiable? = cusp / corner

I. $x = 2$ II. $x = 3$ III. $x = 5$

A. II only
B. I only
C. II and III only
D. I and II only
E. III only



Graph of f

FREE RESPONSE

The table below shows values of differentiable functions, $f(x)$ and $g(x)$, and their derivatives at selected values of x. Use the table of values below to answer each of the questions below.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	3	−1	2	5
1	3	2	3	−3
2	5	3	1	−2
3	10	4	0	−1

a. What does the value of $f'(1.5)$ represent in terms of the graph of $f(x)$? Approximate the value of $f'(1.5)$ and explain why your approximation is a good approximation. Slope between (1, 3) & (2, 5) = 2

b. If $B(x) = \sqrt{g(x)}$, what is the equation of the tangent line drawn to $B(x)$ when $x = 1$?

c. If $A(x) = \ln(f(x))$, what is the value of $A'(2)$? What does this result say about the behavior of the graph of $A(x)$ when $x = 2$? Give a reason for your answer.

b. $B(1) = \sqrt{g(1)} = \sqrt{3}$
$B'(x) = \frac{1}{2}[g(x)]^{-1/2} \cdot g'(x)$
$B'(1) = \frac{1}{2}(3)^{-1/2} \cdot -3 = -\frac{3}{2\sqrt{3}}$ or $-\frac{\sqrt{3}}{2}$

$y - \sqrt{3} = -\frac{\sqrt{3}}{2}(x-1)$

c. $A'(x) = \frac{1}{f(x)} \cdot f'(x)$
$A'(2) = \frac{1}{5} \cdot 3 = \frac{3}{5}$
graph is increasing since A' is positive.